

LUYỆN THI VÀO 10 – MÔN TOÁN
 GIÁO VIÊN: NGUYỄN THÀNH LONG
 CÁC KỸ THUẬT SO SÁNH CĂN BẬC HAI – ĐÁP ÁN

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Bài 1: So sánh các số sau:

a) $\sqrt{11} - \sqrt{3}$ và 2.

b) $\sqrt{3} + 2$ và $\sqrt{2} + \sqrt{6}$

c) $\sqrt{2003} + \sqrt{2005}$ và $2\sqrt{2004}$

d) $\sqrt{8} + \sqrt{5}$ và $\sqrt{7} + \sqrt{6}$

Bài giải:

a) Ta có: $(\sqrt{11} - \sqrt{3})^2 = 11 - 2\sqrt{11}\sqrt{3} + 3 = 14 - 2\sqrt{33}$

$2^2 = 4$

Xét hiệu: $11 - 2\sqrt{33} - 4 = 7 - 2\sqrt{33} = \sqrt{49} - \sqrt{4.33} = \sqrt{49} - \sqrt{132} < 0$

$\Rightarrow (\sqrt{11} - \sqrt{3})^2 < 2^2$

$\Rightarrow \sqrt{11} - \sqrt{3} < 2$

Vậy $\sqrt{11} - \sqrt{3} < 2$

b) Ta có: $(\sqrt{3} + 2)^2 = 3 + 2\sqrt{3}\cdot 2 + 2^2 = 3 + 4\sqrt{3} + 4 = 7 + 4\sqrt{3}$

$(\sqrt{2} + \sqrt{6})^2 = 2 + 2\sqrt{2}\sqrt{6} + 6 = 8 + 2\sqrt{12} = 8 + 4\sqrt{3}$

Do: $7 + 4\sqrt{3} < 8 + 4\sqrt{3} \Rightarrow (\sqrt{3} + 2)^2 < (\sqrt{2} + \sqrt{6})^2$

$\Rightarrow \sqrt{3} + 2 < \sqrt{2} + \sqrt{6}$

Vậy $\sqrt{3} + 2 < \sqrt{2} + \sqrt{6}$

c) Ta có: $(\sqrt{2003} + \sqrt{2005})^2 = 2003 + 2\sqrt{2003}\sqrt{2005} + 2005 = 4008 + 2\sqrt{2003}\sqrt{2005}$

$(2\sqrt{2004})^2 = 4\cdot 2004 = 8016$

Giả sử: $(\sqrt{2003} + \sqrt{2005})^2 < (2\sqrt{2004})^2$

$$\Rightarrow 4008 + 2\sqrt{2003}\sqrt{2005} < 8016$$

$$\Rightarrow 2\sqrt{2003}\sqrt{2005} < 2 \cdot 2004$$

$$\Rightarrow \sqrt{2003}\sqrt{2005} < 2004$$

$$\Rightarrow (\sqrt{2003}\sqrt{2005})^2 < 2004^2$$

$$\Rightarrow 2003 \cdot 2005 < 2004^2$$

$$\Rightarrow (2004 - 1)(2004 + 1) < 2004^2$$

$$\Rightarrow 2004^2 - 1 < 2004^2 \text{ (luôn đúng)}$$

$$\text{Do đó: } \sqrt{2003} + \sqrt{2005} < 2\sqrt{2004}$$

$$\text{Vậy } \sqrt{2003} + \sqrt{2005} < 2\sqrt{2004}$$

$$\text{d) Ta có: } (\sqrt{8} + \sqrt{5})^2 = 8 + 2\sqrt{8}\sqrt{5} + 5 = 13 + 2\sqrt{40}$$

$$(\sqrt{7} + \sqrt{6})^2 = 7 + 2\sqrt{7}\sqrt{6} + 6 = 13 + 2\sqrt{42}$$

$$\text{Vì } 13 + 2\sqrt{40} < 13 + 2\sqrt{42}$$

$$\Rightarrow (\sqrt{8} + \sqrt{5})^2 < (\sqrt{7} + \sqrt{6})^2$$

$$\Rightarrow \sqrt{8} + \sqrt{5} < \sqrt{7} + \sqrt{6}$$

$$\text{Vậy } \sqrt{8} + \sqrt{5} < \sqrt{7} + \sqrt{6}$$

Bài 2: So sánh các số sau:

$$\text{a) } \sqrt{2000} - \sqrt{1999} \text{ và } \sqrt{2001} - \sqrt{2000}$$

$$\text{b) } \sqrt{13} - \sqrt{12} \text{ và } \sqrt{7} - \sqrt{6}$$

$$\text{c) } 3\sqrt{3} - 3 \text{ và } 2\sqrt{2} - 1$$

Bài giải:

$$\text{a) Giả sử: } \sqrt{2000} - \sqrt{1999} > \sqrt{2001} - \sqrt{2000}$$

$$\Leftrightarrow 2\sqrt{2000} > \sqrt{1999} + \sqrt{2001}$$

$$\Leftrightarrow 4 \cdot 2000 > 1999 + 2\sqrt{1999 \cdot 2001} + 2001$$

$$\Leftrightarrow 4.2000 > 2.2000 + 2.\sqrt{1999.2001}$$

$$\Leftrightarrow 2.2000 > 2.\sqrt{1999.2001}$$

$$\Leftrightarrow 2000 > \sqrt{1999.2001}$$

$$\Leftrightarrow 2000^2 > 1999.2001$$

$$\Leftrightarrow 2000^2 > (2000-1)(2000+1)$$

$$\Leftrightarrow 2000^2 > 2000^2 - 1 \text{ (hiển nhiên đúng)}$$

Vậy $\sqrt{2000} - \sqrt{1999} > \sqrt{2001} - \sqrt{2000}$

b) Giả sử: $\sqrt{13} - \sqrt{12} \geq \sqrt{7} - \sqrt{6}$

$$\Leftrightarrow \sqrt{13} + \sqrt{6} \geq \sqrt{12} + \sqrt{7}$$

$$\Leftrightarrow (\sqrt{13} + \sqrt{6})^2 \geq (\sqrt{12} + \sqrt{7})^2$$

$$\Leftrightarrow 19 + 2\sqrt{13.6} \geq 19 + 2\sqrt{12.7}$$

$$\Leftrightarrow \sqrt{13.6} \geq \sqrt{12.7}$$

$$\Leftrightarrow \sqrt{78} \geq \sqrt{84} \text{ (vô lý)}$$

Vậy $\sqrt{13} - \sqrt{12} < \sqrt{7} - \sqrt{6}$

c) Giả sử: $3\sqrt{3} - 3 \geq 2\sqrt{2} - 1$

$$\Leftrightarrow 3\sqrt{3} - 3 - 2\sqrt{2} + 1 \geq 0$$

$$\Leftrightarrow 3\sqrt{3} - 2\sqrt{2} - 2 \geq 0$$

$$\Leftrightarrow 3\sqrt{3} \geq 2\sqrt{2} + 2$$

$$\Leftrightarrow (3\sqrt{3})^2 \geq (2\sqrt{2} + 2)^2$$

$$\Leftrightarrow 27 \geq 8 + 8\sqrt{2} + 4$$

$$\Leftrightarrow 15 \geq 8\sqrt{2}$$

$$\Leftrightarrow 15^2 \geq (8\sqrt{2})^2$$

$$\Leftrightarrow 225 \geq 128 \text{ (vô lý)}$$

Vậy $3\sqrt{3}-3 > 2\sqrt{2}-1$

NGUYỄN THÀNH LONG