

**LUYỆN THI VÀO 10 – MÔN TOÁN**  
**GIÁO VIÊN: NGUYỄN THÀNH LONG**  
**RÚT GỌN CĂN THỨC KHÔNG CHỨA BIẾN (PHẦN 2) – ĐÁP ÁN**

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**Bài 1:** Thực hiện phép tính

a)  $2\sqrt{18} - 7\sqrt{2} + \sqrt{162}$

b)  $2\sqrt{2}(\sqrt{3} - 2) + (1 + 2\sqrt{2})^2 - 2\sqrt{6}$

c)  $(2\sqrt{8} + 3\sqrt{5} - 7\sqrt{2})(\sqrt{72} - 5\sqrt{20} - 2\sqrt{2})$

d)  $(\sqrt{6} + \sqrt{5})^2 - \sqrt{120}$

**Bài giải:**

a)  $2\sqrt{18} - 7\sqrt{2} + \sqrt{162} = 2.3\sqrt{2} - 7\sqrt{2} + 9\sqrt{2} = 8\sqrt{2}$

b)  $2\sqrt{2}(\sqrt{3} - 2) + (1 + 2\sqrt{2})^2 - 2\sqrt{6} = 2\sqrt{6} - 4\sqrt{2} + 1 + 8 + 4\sqrt{2} - 2\sqrt{6} = 9$

c)  $(2\sqrt{8} + 3\sqrt{5} - 7\sqrt{2})(\sqrt{72} - 5\sqrt{20} - 2\sqrt{2})$   
 $= (2.2\sqrt{2} + 3\sqrt{5} - 7\sqrt{2})(6\sqrt{2} - 5.2\sqrt{5} - 2\sqrt{2})$   
 $= (3\sqrt{5} - 3\sqrt{2})(4\sqrt{2} - 10\sqrt{5}) = 3.(\sqrt{5} - \sqrt{2}).2.(2\sqrt{2} - 5\sqrt{5})$

d)  $(\sqrt{6} + \sqrt{5})^2 - \sqrt{120} = 6 + 2\sqrt{30} + 5 - 2\sqrt{30} = 11$

**Nhận xét: Dạng**  $\sqrt{a \pm 2\sqrt{b}} = \sqrt{(\sqrt{m} \pm \sqrt{n})^2}$

Bản chất vấn đề: Ví dụ:  $\sqrt{5 - 2\sqrt{6}}$

Ta nhầm thấy:  $6 = 1.6 = 2.3$ . Lại có:  $2 + 3 = 5$  (còn  $1 + 6 = 7$ )

Vậy ta chọn 2 số 2 và 3 để phân tích:  $5 = 2 + 3 = (\sqrt{2})^2 + (\sqrt{3})^2$  và  $\sqrt{6} = \sqrt{2} \cdot \sqrt{3}$

$$\sqrt{5 - 2\sqrt{6}} = \sqrt{3 - 2\sqrt{3} \cdot \sqrt{2} + 2} = \sqrt{(\sqrt{3} - \sqrt{2})^2} = \sqrt{3} - \sqrt{2}$$

**Bài 2:** Rút gọn các biểu thức sau

a)  $\sqrt{11 - 2\sqrt{10}}$

b)  $\sqrt{27 - 10\sqrt{2}}$

c)  $\sqrt{18 - 6\sqrt{5}}$

d)  $\sqrt{28 - 6\sqrt{3}}$

e)  $\sqrt{46 - 6\sqrt{5}}$

g)  $\sqrt{6 - \sqrt{20}}$

h)  $\sqrt{4+\sqrt{7}}$

i)  $\sqrt{8+\sqrt{15}}$

**Bài giải:**

a)  $\sqrt{11-2\sqrt{10}} = \sqrt{10-2\sqrt{10}+1} = \sqrt{(\sqrt{10}-1)^2} = \sqrt{10}-1$

b)  $\sqrt{27-10\sqrt{2}} = \sqrt{25-2.5.\sqrt{2}+2} = \sqrt{(5-\sqrt{2})^2} = 5-\sqrt{2}$

c)  $\sqrt{18-6\sqrt{5}} = \sqrt{15-2.\sqrt{15}.\sqrt{3}+3} = \sqrt{(\sqrt{5}-\sqrt{3})^2} = \sqrt{5}-\sqrt{3}$

d)  $\sqrt{28-6\sqrt{3}} = \sqrt{27-2.3\sqrt{3}+1} = \sqrt{(3\sqrt{3}-1)^2} = 3\sqrt{3}-1$

e)  $\sqrt{46-6\sqrt{5}} = \sqrt{45-2.3\sqrt{5}+1} = \sqrt{(3\sqrt{5}-1)^2} = 3\sqrt{5}-1$

g)  $\sqrt{6-\sqrt{20}} = \sqrt{6-2\sqrt{5}} = \sqrt{(\sqrt{5}-1)^2} = \sqrt{5}-1$

h)  $\sqrt{4+\sqrt{7}} = \frac{\sqrt{8+2\sqrt{7}}}{\sqrt{2}} = \frac{\sqrt{7+2\sqrt{7}+1}}{\sqrt{2}} = \frac{\sqrt{(\sqrt{7}+1)^2}}{\sqrt{2}} = \frac{\sqrt{7}+1}{\sqrt{2}} = \frac{\sqrt{14}+\sqrt{2}}{2}$

i)  $\sqrt{8+\sqrt{15}} = \frac{\sqrt{16+2\sqrt{15}}}{\sqrt{2}} = \frac{\sqrt{15+2\sqrt{15}+1}}{\sqrt{2}} = \frac{\sqrt{(\sqrt{15}+1)^2}}{\sqrt{2}} = \frac{\sqrt{15}+1}{\sqrt{2}} = \frac{\sqrt{30}+\sqrt{2}}{2}$

**Bài 3:** Thực hiện phép tính

a)  $\sqrt{\sqrt{10}+1}.\sqrt{\sqrt{10}-1}$

b)  $\sqrt{8+2\sqrt{15}} - \sqrt{8-2\sqrt{15}}$

c)  $\sqrt{21+6\sqrt{6}} + \sqrt{21-6\sqrt{6}}$

d)  $\sqrt{8+\sqrt{60}} - \sqrt{8-\sqrt{60}}$

e)  $\sqrt{15-\sqrt{216}} + \sqrt{33-12\sqrt{6}}$

**Bài giải:**

a)  $\sqrt{\sqrt{10}+1}.\sqrt{\sqrt{10}-1} = \sqrt{(\sqrt{10}+1)(\sqrt{10}-1)} = \sqrt{10-1} = \sqrt{9} = 3$

b)  $\sqrt{8+2\sqrt{15}} - \sqrt{8-2\sqrt{15}} = \sqrt{5+2\sqrt{15}+3} - \sqrt{5-2\sqrt{15}+3}$   
 $= \sqrt{(\sqrt{5}+\sqrt{3})^2} - \sqrt{(\sqrt{5}-\sqrt{3})^2} = \sqrt{5}+\sqrt{3} - \sqrt{5}+\sqrt{3} = 2\sqrt{3}$

c)  $\sqrt{21+6\sqrt{6}} + \sqrt{21-6\sqrt{6}} = \sqrt{21+2.3\sqrt{6}} + \sqrt{21-2.3\sqrt{6}}$   
 $= \sqrt{(3\sqrt{2}+\sqrt{3})^2} + \sqrt{(3\sqrt{2}-\sqrt{3})^2} = 3\sqrt{2}+\sqrt{3} + 3\sqrt{2}-\sqrt{3} = 6\sqrt{2}$   
 $= \sqrt{18+2.3\sqrt{2}.\sqrt{3}+3} + \sqrt{18-2.3\sqrt{2}.\sqrt{3}+3}$

d)  $\sqrt{8+\sqrt{60}} - \sqrt{8-\sqrt{60}} = \sqrt{8+2\sqrt{15}} - \sqrt{8-2\sqrt{15}}$

$$= \sqrt{5+2\sqrt{15}+3} - \sqrt{5-2\sqrt{15}+3} = \sqrt{(\sqrt{5}+\sqrt{3})^2} - \sqrt{(\sqrt{5}-\sqrt{3})^2}$$

$$= \sqrt{5} + \sqrt{3} - \sqrt{5} + \sqrt{3} = 2\sqrt{3}$$

$$e) \sqrt{15-\sqrt{216}} + \sqrt{33-12\sqrt{6}} = \sqrt{15-6\sqrt{6}} + \sqrt{33-12\sqrt{6}}$$

$$= \sqrt{9-2.3.\sqrt{6}+6} + \sqrt{24-2.\sqrt{216}+9} = \sqrt{(3-\sqrt{6})^2} + \sqrt{(2\sqrt{6}-3)^2}$$

$$= 3 - \sqrt{6} + 2\sqrt{6} - 3 = \sqrt{6}$$

**Bài 4:** Rút gọn các biểu thức sau:

$$a) A = \frac{\sqrt{8-\sqrt{15}}}{\sqrt{30}-\sqrt{2}}$$

$$b) B = \frac{\sqrt{8-\sqrt{15}} - \sqrt{8+\sqrt{15}}}{\sqrt{2}}$$

$$c) C = (4+\sqrt{3}).\sqrt{19-8\sqrt{3}}$$

**Bài giải:**

$$a) A = \frac{\sqrt{8-\sqrt{15}}}{\sqrt{30}-\sqrt{2}} = \frac{\sqrt{8-\sqrt{15}}}{\sqrt{2}(\sqrt{15}-1)} = \frac{\sqrt{16-2\sqrt{15}}}{2(\sqrt{15}-1)} = \frac{\sqrt{(\sqrt{15}-1)^2}}{2(\sqrt{15}-1)} = \frac{\sqrt{15}-1}{2(\sqrt{15}-1)} = \frac{1}{2}$$

$$b) B = \frac{\sqrt{8-\sqrt{15}} - \sqrt{8+\sqrt{15}}}{\sqrt{2}} = \frac{\sqrt{16-2\sqrt{15}} - \sqrt{16+2\sqrt{15}}}{2} = \frac{\sqrt{(\sqrt{15}-1)^2} - \sqrt{(\sqrt{15}+1)^2}}{2}$$

$$= \frac{\sqrt{15}-1 - \sqrt{15}-1}{2} = \frac{-2}{2} = -1$$

$$c) C = (4+\sqrt{3}).\sqrt{19-8\sqrt{3}} = (4+\sqrt{3}).\sqrt{16-2.4.\sqrt{3}+3} = (4+\sqrt{3}).\sqrt{(4-\sqrt{3})^2}$$

$$= (4+\sqrt{3}).(4-\sqrt{3}) = 16-3 = 13$$

**Bài 5:** Rút gọn các biểu thức sau:

$$a) \sqrt{4+\sqrt{7}} - \sqrt{4-\sqrt{7}} - \sqrt{2}$$

$$b) \sqrt{13+30\sqrt{2+\sqrt{9+4\sqrt{2}}}}$$

$$c) \sqrt{\sqrt{3}-\sqrt{1-\sqrt{21-12\sqrt{3}}}}$$

$$d) \sqrt{6+2\sqrt{5-\sqrt{13+\sqrt{48}}}}$$

**Bài giải:**

$$\begin{aligned} \text{a) } \sqrt{4+\sqrt{7}} - \sqrt{4-\sqrt{7}} - \sqrt{2} &= \frac{\sqrt{8+2\sqrt{7}}}{\sqrt{2}} - \frac{\sqrt{8-2\sqrt{7}}}{\sqrt{2}} - \sqrt{2} \\ &= \frac{\sqrt{(\sqrt{7}+1)^2}}{\sqrt{2}} - \frac{\sqrt{(\sqrt{7}-1)^2}}{\sqrt{2}} - \sqrt{2} = \frac{\sqrt{7}+1}{\sqrt{2}} - \frac{\sqrt{7}-1}{\sqrt{2}} - \sqrt{2} \\ &= \frac{\sqrt{7}+1-\sqrt{7}+1}{\sqrt{2}} - \sqrt{2} = \sqrt{2} - \sqrt{2} = 0 \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{13+30\sqrt{2}+\sqrt{9+4\sqrt{2}}} &= \sqrt{13+30\sqrt{2}+\sqrt{8+2.2\sqrt{2}+1}} \\ &= \sqrt{13+30\sqrt{2}+(2\sqrt{2}+1)^2} = \sqrt{13+30\sqrt{2}+2\sqrt{2}+1} \\ &= \sqrt{13+30\sqrt{3}+2\sqrt{2}} = \sqrt{13+30\sqrt{2}+2\sqrt{2}+1} = \sqrt{13+30\sqrt{(\sqrt{2}+1)^2}} \\ &= \sqrt{13+30(\sqrt{2}+1)} = \sqrt{13+30\sqrt{2}+30} = \sqrt{43+30\sqrt{2}} \\ &= \sqrt{25+2.5.3\sqrt{2}+18} = \sqrt{(5+3\sqrt{2})^2} = 5+3\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt{\sqrt{3}-\sqrt{1-\sqrt{21-12\sqrt{3}}}} &= \sqrt{\sqrt{3}-\sqrt{1-\sqrt{21-2.3.2\sqrt{3}}}} \\ &= \sqrt{\sqrt{3}-\sqrt{1-\sqrt{12-2.3.2\sqrt{3}+9}}} = \sqrt{\sqrt{3}-\sqrt{1-\sqrt{(2\sqrt{3}-3)^2}}} \\ &= \sqrt{\sqrt{3}-\sqrt{1-2\sqrt{3}+3}} = \sqrt{\sqrt{3}-\sqrt{4-2\sqrt{3}}} = \sqrt{\sqrt{3}-\sqrt{(\sqrt{3}-1)^2}} \\ &= \sqrt{\sqrt{3}-(\sqrt{3}-1)} = \sqrt{1} = 1 \end{aligned}$$

$$\begin{aligned} \text{d) } \sqrt{6+2\sqrt{5}-\sqrt{13+\sqrt{48}}} &= \sqrt{6+2\sqrt{5}-\sqrt{13+4\sqrt{3}}} \\ &= \sqrt{6+2\sqrt{5}-\sqrt{12+2.2\sqrt{3}+1}} = \sqrt{6+2\sqrt{5}-\sqrt{(2\sqrt{3}+1)^2}} \\ &= \sqrt{6+2\sqrt{5}-2\sqrt{3}-1} = \sqrt{6+2\sqrt{4-2\sqrt{3}}} = \sqrt{6+2\sqrt{(\sqrt{3}-1)^2}} \\ &= \sqrt{6+2(\sqrt{3}-1)} = \sqrt{6+2\sqrt{3}-2} = \sqrt{4+2\sqrt{3}} = \sqrt{(\sqrt{3}+1)^2} = \sqrt{3}+1 \end{aligned}$$

**Bài 6:** Rút gọn các biểu thức sau:

$$\text{a) } A = (4+\sqrt{15})(\sqrt{5}-\sqrt{3})\sqrt{4-\sqrt{15}}$$

$$\text{b) } B = \sqrt{3-\sqrt{5}}(\sqrt{10}-\sqrt{2})(3+\sqrt{5})$$

$$c) C = (\sqrt{14} - \sqrt{10})(6 - \sqrt{35})\sqrt{6 + \sqrt{35}}$$

**Bài giải:**

$$\begin{aligned} a) A &= (4 + \sqrt{15})(\sqrt{5} - \sqrt{3})\sqrt{4 - \sqrt{15}} = \frac{1}{\sqrt{2}} \cdot (4 + \sqrt{15})(\sqrt{5} - \sqrt{3})\sqrt{8 - 2\sqrt{15}} \\ &= \frac{1}{\sqrt{2}}(4 + \sqrt{15})(\sqrt{5} - \sqrt{3}) \cdot \sqrt{(\sqrt{5} - \sqrt{3})^2} = \frac{1}{\sqrt{2}}(4 + \sqrt{15})(\sqrt{5} - \sqrt{3})^2 \\ &= \frac{1}{\sqrt{2}}(4 + \sqrt{15})(8 - 2\sqrt{15}) = \frac{1}{\sqrt{2}}(4 + \sqrt{15}) \cdot 2(4 - \sqrt{15}) = \sqrt{2}(16 - 15) = \sqrt{2} \end{aligned}$$

$$\begin{aligned} b) B &= \sqrt{3 - \sqrt{5}}(\sqrt{10} - \sqrt{2})(3 + \sqrt{5}) = \sqrt{6 - 2\sqrt{5}}(\sqrt{5} - 1)(3 + \sqrt{5}) \\ &= \sqrt{(\sqrt{5} - 1)^2}(\sqrt{5} - 1)(3 + \sqrt{5}) = (\sqrt{5} - 1)^2(3 + \sqrt{5}) \\ &= (6 - 2\sqrt{5})(3 + \sqrt{5}) = 2 \cdot (3 - \sqrt{5})(3 + \sqrt{5}) \\ &= 2 \cdot (6 - 5) = 2 \cdot 1 = 2 \end{aligned}$$

$$\begin{aligned} c) C &= (\sqrt{14} - \sqrt{10})(6 - \sqrt{35})\sqrt{6 + \sqrt{35}} = (\sqrt{7} - \sqrt{5})(6 - \sqrt{35})\sqrt{12 + 2\sqrt{35}} \\ &= (\sqrt{7} - \sqrt{5})(6 - \sqrt{35})\sqrt{(\sqrt{7} + \sqrt{5})^2} = (\sqrt{7} - \sqrt{5})(6 - \sqrt{35})(\sqrt{7} + \sqrt{5}) \\ &= (7 - 5)(6 - \sqrt{35}) = 2(6 - \sqrt{35}) = 12 - 2\sqrt{35} \end{aligned}$$

**Bài 7:** Rút gọn các biểu thức sau:

$$a) A = \frac{1}{2 - \sqrt{3}} - \frac{1}{2 + \sqrt{3}}$$

$$b) B = \frac{1}{\sqrt{5} + \sqrt{3}} - \frac{1}{\sqrt{5} - \sqrt{3}}$$

$$c) C = \frac{\sqrt{4 - 2\sqrt{3}}}{\sqrt{6} - \sqrt{2}}$$

$$d) D = \frac{1}{2 + \sqrt{3}} + \frac{\sqrt{2}}{\sqrt{6}} - \frac{2}{3 + \sqrt{3}}$$

**Bài giải:**

$$\begin{aligned} a) A &= \frac{1}{2 - \sqrt{3}} - \frac{1}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{(2 - \sqrt{3})(2 + \sqrt{3})} - \frac{2 - \sqrt{3}}{(2 - \sqrt{3})(2 + \sqrt{3})} \\ &= \frac{2 + \sqrt{3} - 2 + \sqrt{3}}{4 - 3} = 2\sqrt{3} \end{aligned}$$

$$b) A = \frac{1}{\sqrt{5+\sqrt{3}}} - \frac{1}{\sqrt{5-\sqrt{3}}} = \frac{\sqrt{5-\sqrt{3}}}{5-3} - \frac{\sqrt{5+\sqrt{3}}}{5-3} = \frac{\sqrt{5-\sqrt{3}} - \sqrt{5+\sqrt{3}}}{2} = -\sqrt{3}$$

$$c) B = \frac{\sqrt{4-2\sqrt{3}}}{\sqrt{6-\sqrt{2}}} = \frac{\sqrt{3-2\sqrt{3}+1}}{\sqrt{2}(\sqrt{3}-1)} = \frac{\sqrt{(\sqrt{3}-1)^2}}{\sqrt{2}(\sqrt{3}-1)} = \frac{\sqrt{3}-1}{\sqrt{2}(\sqrt{3}-1)} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$d) D = \frac{1}{2+\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{6}} - \frac{2}{3+\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} + \frac{\sqrt{12}}{6} - \frac{2(3-\sqrt{3})}{9-3}$$

$$= 2 - \sqrt{3} + \frac{2\sqrt{3}}{6} - \frac{6-2\sqrt{3}}{6} = 2 - \sqrt{3} + \frac{4\sqrt{3}-6}{6} = 2 - \sqrt{3} + \frac{2}{3}\sqrt{3} - 1$$

$$= 1 - \frac{\sqrt{3}}{3} = \frac{3-\sqrt{3}}{3}$$

**Bài 8:** Rút gọn các biểu thức sau:

$$a) A = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} + \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$$

$$b) B = \left(2 + \frac{3+\sqrt{3}}{\sqrt{3}+1}\right) \cdot \left(2 - \frac{3-\sqrt{3}}{\sqrt{3}-1}\right)$$

$$c) C = \frac{(2+\sqrt{3})\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = \frac{(2+\sqrt{3})(2-\sqrt{3})}{\sqrt{4-3}}$$

$$d) D = \left(\frac{1}{\sqrt{7-\sqrt{24}+1}} - \frac{1}{\sqrt{7+\sqrt{24}-1}} + 1\right) : (\sqrt{3}-\sqrt{2})$$

**Bài giải:**

$$a) A = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} + \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})}} + \sqrt{\frac{(2+\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})}}$$

$$= \sqrt{\frac{(2-\sqrt{3})^2}{4-3}} + \sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$$

$$b) B = \left(2 + \frac{3+\sqrt{3}}{\sqrt{3}+1}\right) \cdot \left(2 - \frac{3-\sqrt{3}}{\sqrt{3}-1}\right) = (2+\sqrt{3})(2-\sqrt{3}) = 4-3=1$$

$$c) C = \frac{(2+\sqrt{3})\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = \frac{(2+\sqrt{3})(2-\sqrt{3})}{\sqrt{4-3}} = (2+\sqrt{3})(2-\sqrt{3}) = 4-3=1$$

$$d) D = \left(\frac{1}{\sqrt{7-\sqrt{24}+1}} - \frac{1}{\sqrt{7+\sqrt{24}-1}} + 1\right) : (\sqrt{3}-\sqrt{2})$$

$$\begin{aligned}
 &= \left( \frac{1}{\sqrt{7-2\sqrt{6}+1}} - \frac{1}{\sqrt{7+2\sqrt{6}-1}} + 1 \right) : (\sqrt{3}-\sqrt{2}) \\
 &= \left( \frac{1}{\sqrt{(\sqrt{6}-1)^2+1}} - \frac{1}{\sqrt{(\sqrt{6}+1)^2-1}} + 1 \right) : (\sqrt{3}-\sqrt{2}) \\
 &= \left( \frac{1}{\sqrt{6-1+1}} - \frac{1}{\sqrt{6+1-1}} + 1 \right) : (\sqrt{3}-\sqrt{2}) \\
 &= \left( \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} + 1 \right) : (\sqrt{3}-\sqrt{2}) = \frac{1}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3}+\sqrt{2}
 \end{aligned}$$

**Bài 9:** Rút gọn các biểu thức sau

a)  $A = \sqrt{4+\sqrt{10+2\sqrt{5}}} + \sqrt{4-\sqrt{10+2\sqrt{5}}}$

b)  $B = \frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}-1} - \frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}+1}$

c)  $C = \frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}} + \frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$

d)  $D = \frac{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+\sqrt{16}}{\sqrt{2}+\sqrt{3}+\sqrt{4}}$

**Bài giải:**

a)  $A = \sqrt{4+\sqrt{10+2\sqrt{5}}} + \sqrt{4-\sqrt{10+2\sqrt{5}}}$

Ta có:

$$A^2 = 4 + \sqrt{10+2\sqrt{5}} + 4 - \sqrt{10+2\sqrt{5}} + 2\sqrt{(4+\sqrt{10+2\sqrt{5}})(4-\sqrt{10+2\sqrt{5}})}$$

$$= 8 + 2\sqrt{16 - (10+2\sqrt{5})} = 8 + 2\sqrt{6-2\sqrt{5}} = 8 + 2\sqrt{5-2\sqrt{5}+1}$$

$$= 8 + 2\sqrt{(\sqrt{5}-1)^2} = 8 + 2(\sqrt{5}-1) = 8 + 2\sqrt{5} - 2 = 6 + 2\sqrt{5}$$

$$\Rightarrow A = \sqrt{6+2\sqrt{5}} = \sqrt{5+2\sqrt{5}+1} = \sqrt{(\sqrt{5}+1)^2} = \sqrt{5}+1$$

b)  $B = \frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}-1} - \frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}+1}$

$$= \frac{\sqrt{3+\sqrt{3}+\sqrt{3}}}{(\sqrt{\sqrt{3}+1}-1)(\sqrt{\sqrt{3}+1}+1)} - \frac{\sqrt{3+\sqrt{3}-\sqrt{3}}}{(\sqrt{\sqrt{3}+1}-1)(\sqrt{\sqrt{3}+1}+1)}$$

$$= \frac{2\sqrt{3}}{\sqrt{3+1}-1} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$$

$$\begin{aligned} \text{c) } C &= \frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}} + \frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}} \\ &= \frac{2\sqrt{2}+\sqrt{6}}{2+\sqrt{4+2\sqrt{3}}} + \frac{2\sqrt{2}-\sqrt{6}}{2-\sqrt{4-2\sqrt{3}}} = \frac{2\sqrt{2}+\sqrt{6}}{2+\sqrt{(\sqrt{3}+1)^2}} + \frac{2\sqrt{2}-\sqrt{6}}{2-\sqrt{(\sqrt{3}-1)^2}} \\ &= \frac{2\sqrt{2}+\sqrt{6}}{2+\sqrt{3}+1} + \frac{2\sqrt{2}-\sqrt{6}}{2-\sqrt{3}+1} = \frac{2\sqrt{2}+\sqrt{6}}{3+\sqrt{3}} + \frac{2\sqrt{2}-\sqrt{6}}{3-\sqrt{3}} \\ &= \frac{(2\sqrt{2}+\sqrt{6})(3-\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} + \frac{(2\sqrt{2}-\sqrt{6})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} \\ &= \frac{6\sqrt{2}-2\sqrt{6}+3\sqrt{6}-3\sqrt{2}}{9-3} + \frac{6\sqrt{2}+2\sqrt{6}-3\sqrt{6}-3\sqrt{2}}{9-3} \\ &= \frac{3\sqrt{2}+\sqrt{6}}{6} + \frac{3\sqrt{2}-\sqrt{6}}{6} = \frac{6\sqrt{2}}{6} = \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{d) } D &= \frac{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+\sqrt{16}}{\sqrt{2}+\sqrt{3}+\sqrt{4}} = \frac{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+4}{\sqrt{2}+\sqrt{3}+\sqrt{4}} \\ &= \frac{(\sqrt{2}+\sqrt{3}+2)+(2+\sqrt{6}+\sqrt{8})}{\sqrt{2}+\sqrt{3}+\sqrt{4}} = \frac{(\sqrt{2}+\sqrt{3}+\sqrt{4})+(\sqrt{4}+\sqrt{6}+\sqrt{8})}{\sqrt{2}+\sqrt{3}+\sqrt{4}} \\ &= \frac{(\sqrt{2}+\sqrt{3}+\sqrt{4})+\sqrt{2}\cdot(\sqrt{2}+\sqrt{3}+\sqrt{4})}{\sqrt{2}+\sqrt{3}+\sqrt{4}} = \frac{(\sqrt{2}+\sqrt{3}+\sqrt{4})(\sqrt{2}+1)}{\sqrt{2}+\sqrt{3}+\sqrt{4}} \\ &= \sqrt{2}+1 \end{aligned}$$

**Bài 10:** Rút gọn các biểu thức sau:

$$\text{a) } A = \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{n-1}+\sqrt{n}}$$

$$\text{b) } B = \frac{1}{\sqrt{2}-\sqrt{3}} - \frac{1}{\sqrt{3}-\sqrt{4}} + \frac{1}{\sqrt{4}-\sqrt{5}} - \dots + \frac{1}{\sqrt{2n}-\sqrt{2n+1}}$$

$$\text{c) } C = \frac{1}{2\sqrt{1}+1\sqrt{2}} + \frac{1}{3\sqrt{2}+2\sqrt{3}} + \frac{1}{4\sqrt{3}+3\sqrt{4}} + \dots + \frac{1}{100\sqrt{99}+99\sqrt{100}}$$

**Bài giải:**



$$\begin{aligned}
 \text{a) } A &= \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{n-1}+\sqrt{n}} \\
 &= \frac{\sqrt{2}-\sqrt{1}}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \dots + \frac{\sqrt{n}-\sqrt{n-1}}{n-(n-1)} \\
 &= \sqrt{2}-1 + \sqrt{3}-\sqrt{2} + \sqrt{4}-\sqrt{3} + \dots + \sqrt{n}-\sqrt{n-1} \\
 &= \sqrt{n}-1
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } B &= \frac{1}{\sqrt{2}-\sqrt{3}} - \frac{1}{\sqrt{3}-\sqrt{4}} + \frac{1}{\sqrt{4}-\sqrt{5}} - \dots + \frac{1}{\sqrt{2n}-\sqrt{2n+1}} \\
 &= \frac{\sqrt{2}+\sqrt{3}}{2-3} - \frac{\sqrt{3}+\sqrt{4}}{3-4} + \dots + \frac{\sqrt{2n}+\sqrt{2n+1}}{2n-(2n+1)} \\
 &= -\sqrt{2}-\sqrt{3} + \sqrt{3}+\sqrt{4} - \dots - \sqrt{2n}-\sqrt{2n+1} \\
 &= -\sqrt{2}-\sqrt{2n+1}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } C &= \frac{1}{2\sqrt{1}+1\sqrt{2}} + \frac{1}{3\sqrt{2}+2\sqrt{3}} + \frac{1}{4\sqrt{3}+3\sqrt{4}} + \dots + \frac{1}{100\sqrt{99}+99\sqrt{100}} \\
 &= \frac{1}{\sqrt{1.2}(\sqrt{1}+\sqrt{2})} + \frac{1}{\sqrt{2.3}(\sqrt{2}+\sqrt{3})} + \frac{1}{\sqrt{3.4}(\sqrt{3}+\sqrt{4})} + \dots + \frac{1}{\sqrt{99.100}(\sqrt{99}+\sqrt{100})} \\
 &= \frac{\sqrt{2}-\sqrt{1}}{\sqrt{1.2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2.3}} + \frac{\sqrt{4}-\sqrt{3}}{\sqrt{3.4}} + \dots + \frac{\sqrt{100}-\sqrt{99}}{\sqrt{99.100}} \\
 &= 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots + \frac{1}{\sqrt{99}} - \frac{1}{\sqrt{100}} \\
 &= 1 - \frac{1}{\sqrt{100}} = 1 - \frac{1}{10} = \frac{9}{10}
 \end{aligned}$$

NGUYỄN THÀNH LONG