VINA 3 – BỒI DƯỠNG HỌC SINH GIỎI TOÁN 7

GIÁO VIÊN: NGUYỄN THÀNH LONG

MỘT SỐ BÀI TOÁN LIÊN QUAN ĐẾN PHẦN LẺ CỦA SỐ THỰC – ĐÁP ÁN

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Bài 1:

1)Tính giá trị biểu thức: $A=\left\{\frac{1}{4}\right\}+\left\{\frac{5}{4}\right\}+\left\{\frac{9}{4}\right\}+\ldots+\left\{\frac{4n+1}{4}\right\}$, với n là số tự nhiên.

2) Cho $B=\left\{\frac{2}{5}\right\}+\left\{\frac{7}{5}\right\}+\left\{\frac{12}{5}\right\}+\ldots+\left\{\frac{2017}{5}\right\}$. Tính {B}?

Bài 2: Rút gọn biểu thức: $S=\left[\frac{n+1}{2}\right]+\left[\frac{n+2}{2^2}\right]+\left[\frac{n+2^2}{2^3}\right]+\ldots+\left[\frac{n+2^{2017}}{2^{2018}}\right]$.

Bài 3:

1. Chứng minh rằng: $\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=[x]$, với mọi số thực x.

2. Rút gọn biểu thức:

$$M=\left(\left[\frac{x+1}{3}\right]+\left[\frac{3x+1}{3}\right]+\ldots+\left[\frac{3^nx+1}{3}\right]\right)+\left(\left[\frac{x+2}{3}\right]+\left[\frac{3x+2}{3}\right]+\ldots+\left[\frac{3^nx+2}{3}\right]\right).$$

Bài 4: Rút gọn biểu thức:

$$P=\left(\left\{\frac{x}{3}+\frac{1}{3}\right\}+\left\{\frac{x}{3^2}+\frac{1}{3}\right\}+\ldots+\left\{\frac{x}{3^n}+\frac{1}{3}\right\}\right)+\left(\left\{\frac{x}{3}+\frac{2}{3}\right\}+\left\{\frac{x}{3^2}+\frac{2}{3}\right\}+\ldots+\left\{\frac{x}{3^n}+\frac{2}{3}\right\}\right)$$

Lời giải:

Bài 1:

1) Ta có $\left\{\frac{4n+1}{4}\right\}=\left\{n+\frac{1}{4}\right\}=\left\{\frac{1}{4}\right\}$ (do n là số tự nhiên).

Số số hạng của tổng là: $\frac{(4n+1)-1}{4}+1=n+1$.

Do đó $A=\frac{1}{4}.(n+1)=\frac{n+1}{4}$.

2) Các số hạng của B có dạng $\left\{\frac{5k+2}{5}\right\}=\left\{k+\frac{2}{5}\right\}=\frac{2}{5}$, với k là số tự nhiên.

Số số hạng của tổng là: $\frac{2017-2}{5}+1=404$.

Do đó $B=\frac{2}{5}.404=\frac{808}{5}=161+\frac{3}{5}$. Suy ra $\{B\}=\frac{3}{5}$.

Bài 2: Theo VD2, ta có: $\left[\frac{x}{2}\right]+\left[\frac{x+1}{2}\right]=[x]\Rightarrow\left[\frac{x+1}{2}\right]=[x]-\left[\frac{x}{2}\right]$.

Do đó ta có

$$S=\left[\frac{n+1}{2}\right]+\left[\frac{n+2}{2^2}\right]+\left[\frac{n+2^2}{2^3}\right]+\ldots+\left[\frac{n+2^{2017}}{2^{2018}}\right]$$

$$=\left[\frac{n}{2}+\frac{1}{2}\right]+\left[\frac{n}{2^2}+\frac{1}{2}\right]+\left[\frac{n}{2^3}+\frac{1}{2}\right]+\ldots+\left[\frac{n}{2^{2018}}+\frac{1}{2}\right]$$

$$=[n]-\left[\frac{n}{2}\right]+\left[\frac{n}{2}\right]-\left[\frac{n}{2^2}\right]+\left[\frac{n}{2^2}\right]-\left[\frac{n}{2^3}\right]+\ldots+\left[\frac{n}{2^{2017}}\right]-\left[\frac{n}{2^{2018}}\right]$$

$$=[n]-\left[\frac{n}{2^{2018}}\right].$$

Bài 3:

1) Đặt $[x]=a\Rightarrow a\le x<a+1$.

Khi đó ta có:

$$\left[\frac{a}{3}\right]+\left[\frac{a+1}{3}\right]+\left[\frac{a+2}{3}\right]\le\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]<\left[\frac{a+1}{3}\right]+\left[\frac{a+2}{3}\right]+\left[\frac{a+3}{3}\right]\quad(1).$$

Đến đây ta xét 3 trường hợp:

TH1: a=3k (k là số nguyên), khi đó thay vào (1) ta được:

$$3k\le\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]<3k+1\Rightarrow\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=3k=a.$$

TH2: a=3k+1 (k là số nguyên), khi đó thay vào (1) ta được:

$$3k+1\le\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]<3k+2\Rightarrow\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=3k+1=a.$$

TH3: a=3k+2 (k là số nguyên), khi đó thay vào (1) ta được:

$$3k+2\le\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]<3k+3\Rightarrow\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=3k+2=a.$$

Do đó trong mọi trường hợp, ta đều có $\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=a=[x]$, do đó ta có điều phải chứng minh.

2) Theo phần trên, ta có:

$$\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=[x]\Rightarrow\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=[x]-\left[\frac{x}{3}\right]$$

Do đó ta có

$$M=\left(\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]\right)+\left(\left[\frac{3x+1}{3}\right]+\left[\frac{3x+2}{3}\right]\right)+\ldots+\left(\left[\frac{3^n x+1}{3}\right]+\left[\frac{3^n x+2}{3}\right]\right)$$

$$=\left(\left[\frac{x}{3}+\frac{1}{3}\right]+\left[\frac{x}{3}+\frac{2}{3}\right]\right)+\left(\left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right]\right)+\ldots+\left(\left[3^{n-1}x+\frac{1}{3}\right]+\left[3^{n-1}x+\frac{2}{3}\right]\right)$$

$$=[x]-\left[\frac{x}{3}\right]+[3x]-[x]+\ldots+\left[3^n x\right]-\left[3^{n-1}x\right]$$

$$=\left[3^n x\right]-\left[\frac{x}{3}\right].$$

Bài 4: Theo bài trên, ta có: $\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=[x]$

Suy ra:

$$\frac{x}{3}-\left\{\frac{x}{3}\right\}+\frac{x+1}{3}-\left\{\frac{x+1}{3}\right\}+\frac{x+2}{3}-\left\{\frac{x+2}{3}\right\}=x-\{x\}\Rightarrow x+1-\left(\left\{\frac{x}{3}\right\}+\left\{\frac{x+1}{3}\right\}+\left\{\frac{x+2}{3}\right\}\right)=x-\{x\}$$

$$\Rightarrow\{x\}+1=\left\{\frac{x}{3}\right\}+\left\{\frac{x+1}{3}\right\}+\left\{\frac{x+2}{3}\right\}\Rightarrow\{x\}-\left\{\frac{x}{3}\right\}+1=\left\{\frac{x}{3}+\frac{1}{3}\right\}+\left\{\frac{x}{3}+\frac{2}{3}\right\}.$$

Số số hạng của P là n nên ta có:

$$P=\left(\left\{\frac{x}{3}+\frac{1}{3}\right\}+\left\{\frac{x}{3}+\frac{2}{3}\right\}\right)+\left(\left\{\frac{x}{3^2}+\frac{1}{3}\right\}+\left\{\frac{x}{3^2}+\frac{2}{3}\right\}\right)+\ldots+\left(\left\{\frac{x}{3^n}+\frac{1}{3}\right\}+\left\{\frac{x}{3^n}+\frac{2}{3}\right\}\right)$$

$$=\left(\{x\}-\left\{\frac{x}{3}\right\}+1\right)+\left(\left\{\frac{x}{3}\right\}-\left\{\frac{x}{3^2}\right\}+1\right)+\ldots+\left(\left\{\frac{x}{3^{n-1}}\right\}-\left\{\frac{x}{3^n}\right\}+1\right)$$

$$=\{x\}-\left\{\frac{x}{3^n}\right\}+n.$$

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