VINA 3 – BỒI DƯỠNG HỌC SINH GIỎI TOÁN 8

GIÁO VIÊN: NGUYỄN THÀNH LONG

CÁC BÀI TOÁN VỀ BIỂU THỨC HỮU TỈ (PHẦN 3)

Bài 1: Rút gọn các biểu thức:

a) $A=\frac{3}{(1.2)^2}+\frac{5}{(2.3)^2}+...+\frac{2n+1}{\left[n(n+1)\right]^2}$

b) $B=\left(1-\frac{1}{2^2}\right).\left(1-\frac{1}{3^2}\right).\left(1-\frac{1}{4^2}\right)......\left(1-\frac{1}{n^2}\right)$

c) $C=\frac{150}{5.8}+\frac{150}{8.11}+\frac{150}{11.14}+...+\frac{150}{47.50}$

d) $D=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(n-1)n(n+1)}$

Bài giải:

Phương pháp: Xuất phát từ hạng tử cuối để tìm ra quy luật

Ta có: $\frac{2n+1}{\left[n(n+1)\right]^2}=\frac{2n+1}{n^2(n+1)^2}=\frac{1}{n^2}-\frac{1}{(n+1)^2}$ nên

$$A=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{3^2}-...-\frac{1}{n^2}+\frac{1}{n^2}-\frac{1}{(n+1)^2}=\frac{1}{1}-\frac{1}{(n+1)^2}=\frac{n(n+1)}{(n+1)^2}$$

b)Ta có: $1-\frac{1}{k^2}=\frac{k^2-1}{k^2}=\frac{(k+1)(k-1)}{k^2}$ nên

$$B=\frac{1.3}{2^2}.\frac{2.4}{3^2}.\frac{3.5}{4^2}...\frac{(n-1)(n+1)}{n^2}=\frac{1.3.2.4...(n-1)(n+1)}{2^2.3^2.4^2...n^2}=\frac{1.2.3...(n-1)}{2.3.4...(n-1)n}.\frac{3.4.5...(n+1)}{2.3.4...n}=\frac{1}{n}.\frac{n+1}{2}=\frac{n+1}{2n}$$

c)

$$C=150.\frac{1}{3}.\left(\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+...+\frac{1}{47}-\frac{1}{50}\right)$$

$$=50.\left(\frac{1}{5}-\frac{1}{50}\right)=50.\frac{9}{10}=45$$

d) $D=\frac{1}{2}.\left(\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{2.3}-\frac{1}{3.4}+...+\frac{1}{(n-1)n}-\frac{1}{n(n+1)}\right)=\frac{1}{2}\left[\frac{1}{1.2}-\frac{1}{n(n+1)}\right]=\frac{(n-1)(n+2)}{4n(n+1)}$

Bài 2:

a)Cho $A=\frac{n-1}{1}+\frac{n-2}{2}+...+\frac{2}{n-2}+\frac{1}{n-1};B=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n}$. Tính $\frac{A}{B}$?

b) $A=\frac{1}{1.(2n-1)}+\frac{1}{3.(2n-3)}+...+\frac{1}{(2n-3).3}+\frac{1}{(2n-1).1};B=1+\frac{1}{3}+...+\frac{1}{2n-1}$

Tính A : B ?

Bài giải:

a)Ta có:

$$A=\left(\frac{n}{1}+\frac{n}{2}+...+\frac{n}{n-2}+\frac{n}{n-1}\right)-\left(\underbrace{1+1+...+1}_{n-1}\right)=n\left(\frac{1}{1}+\frac{1}{2}+...+\frac{1}{n-2}+\frac{1}{n-1}\right)-(n-1)$$

$$=n\left(\frac{1}{1}+\frac{1}{2}+...+\frac{1}{n-2}+\frac{1}{n-1}\right)+1=n\left(\frac{1}{2}+...+\frac{1}{n-2}+\frac{1}{n-1}\right)=nB=>\frac{A}{B}=n$$

b)

$$A=\frac{1}{2n}\left[\left(1+\frac{1}{2n-1}\right)+\left(\frac{1}{3}+\frac{1}{2n-3}\right)+...+\left(\frac{1}{2n-3}+\frac{1}{3}\right)+\left(\frac{1}{2n-1}+1\right)\right]$$

$$=\frac{1}{2n}.2.\left(1+\frac{1}{3}+...+\frac{1}{2n-1}+\frac{1}{2n-3}\right)=\frac{1}{2n}.2.B=>\frac{A}{B}=\frac{1}{n}$$

VINASTUDY.VN