

VINA 3 – BỒI DƯỠNG HỌC SINH GIỎI TOÁN 6

GIÁO VIÊN: NGUYỄN THÀNH LONG

MỘT SỐ BÀI TOÁN VỀ DÃY PHÂN SỐ CÓ QUY LUẬT (PHẦN 2) – ĐÁP ÁN

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Bài 1: Tính $\frac{C}{D}$. Biết $C = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{48} + \frac{1}{49} + \frac{1}{50}$ và $D = \frac{1}{49} + \frac{2}{48} + \frac{3}{47} + \dots + \frac{48}{2} + \frac{49}{1}$

Bài giải:

$$D = \frac{1}{49} + \frac{2}{48} + \frac{3}{47} + \dots + \frac{48}{2} + \frac{49}{1}$$

$$D = \left(1 + \frac{1}{49}\right) + \left(1 + \frac{1}{48}\right) + \left(1 + \frac{1}{47}\right) + \dots + \left(1 + \frac{1}{48}\right) + 1$$

$$D = \frac{50}{49} + \frac{50}{48} + \frac{50}{47} + \dots + \frac{50}{2} + \frac{50}{50}$$

$$D = 50 \cdot \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{50}\right) = 50C$$

$$\text{Do đó: } \frac{C}{D} = \frac{1}{50}$$

Bài 2: Tính giá trị của biểu thức: $C = \frac{\frac{2013}{2} + \frac{2013}{3} + \frac{2013}{4} + \dots + \frac{2013}{2014}}{\frac{2013}{1} + \frac{2012}{2} + \frac{2011}{3} + \dots + \frac{1}{2013}}$

Bài giải:

$$C = \frac{\frac{2013}{2} + \frac{2013}{3} + \frac{2013}{4} + \dots + \frac{2013}{2014}}{\frac{2013}{1} + \frac{2012}{2} + \frac{2011}{3} + \dots + \frac{1}{2013}}$$

$$\text{Tử số: } \frac{2013}{2} + \frac{2013}{3} + \frac{2013}{4} + \dots + \frac{2013}{2014} = 20013 \cdot \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2014}\right)$$

$$\text{Mẫu số: } \frac{2013}{1} + \frac{2012}{2} + \frac{2011}{3} + \dots + \frac{1}{2013} = 1 + \left(1 + \frac{2012}{2}\right) + \left(1 + \frac{2011}{3}\right) + \dots + \left(1 + \frac{1}{2013}\right)$$

$$= \frac{2014}{2014} + \frac{2014}{2} + \frac{2014}{3} + \dots + \frac{2014}{2013} = 2014 \cdot \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2013} + \frac{1}{2014}\right)$$

$$\text{Vậy } C = \frac{2013}{2014}$$

Bài 3: Rút gọn: $\left(\frac{1}{2012} + \frac{2}{2011} + \frac{3}{2010} + \dots + \frac{2011}{2} + \frac{2012}{1}\right) : \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2013}\right)$

Bài giải:

$$A = \left(\frac{1}{2012} + \frac{2}{2011} + \frac{3}{2010} + \dots + \frac{2011}{2} + \frac{2012}{1}\right) : \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2013}\right)$$

$$\text{Số bị chia: } B = \left(\frac{1}{2012} + 1\right) + \left(\frac{2}{2011} + 1\right) + \left(\frac{3}{2010} + 1\right) + \dots + \left(\frac{2011}{2} + 1\right) + 1$$

$$B = \frac{2013}{2012} + \frac{2013}{2011} + \frac{20013}{2010} + \dots + \frac{2013}{2} + \frac{2013}{2013}$$

$$B = 2013 \cdot \left(\frac{1}{2} + \dots + \frac{1}{2010} + \frac{1}{2011} + \frac{1}{2012} + \frac{1}{2013}\right)$$

$$\text{Vậy } A = \left(\frac{1}{2012} + \frac{2}{2011} + \frac{3}{2010} + \dots + \frac{2011}{2} + \frac{2012}{1}\right) : \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2013}\right)$$

$$= 2013 \cdot \left(\frac{1}{2} + \dots + \frac{1}{2010} + \frac{1}{2011} + \frac{1}{2012} + \frac{1}{2013}\right) : \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2013}\right) = 2013$$

Bài 4: Cho $S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2013} - \frac{1}{2014} + \frac{1}{2015}$

$$P = \frac{1}{1008} + \frac{1}{1009} + \frac{1}{1010} + \dots + \frac{1}{2014} + \frac{1}{2015}$$

Tính $(S - P)^{2016}$

Bài giải:

$$S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2013} - \frac{1}{2014} + \frac{1}{2015}$$

$$S = \left(1 + \frac{1}{3} + \dots + \frac{1}{2013} + \frac{1}{2015}\right) - \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2014}\right)$$

$$S = \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2013} + \frac{1}{2014} + \frac{1}{2015}\right) - 2 \cdot \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2014}\right)$$

$$S = \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2013} + \frac{1}{2014} + \frac{1}{2015}\right) - \left(1 + \frac{1}{2} + \dots + \frac{1}{1007}\right)$$

$$S = \frac{1}{1008} + \frac{1}{1009} + \frac{1}{1010} + \dots + \frac{1}{2014} + \frac{1}{2015}$$

$$\text{Mà: } P = \frac{1}{1008} + \frac{1}{1009} + \frac{1}{1010} + \dots + \frac{1}{2014} + \frac{1}{2015}$$

$$\text{Nên: } S - P = 0$$

$$\text{Do đó: } (S - P)^{2016} = 0$$

$$\text{Bài 5: Chứng minh: } \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots + \frac{1}{99.100} = \frac{1}{51} + \frac{1}{52} + \frac{1}{53} + \dots + \frac{1}{100}$$

Bài giải:

$$\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots + \frac{1}{99.100} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100}$$

$$= \left(\frac{1}{1} + \frac{1}{3} + \dots + \frac{1}{99} \right) - \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{100} \right) = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{99} + \frac{1}{100} \right) - 2 \cdot \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{100} \right)$$

$$= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{99} + \frac{1}{100} \right) - \left(1 + \frac{1}{2} + \dots + \frac{1}{50} \right) = \frac{1}{51} + \frac{1}{52} + \dots + \frac{1}{100}$$

$$\text{Vậy } \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots + \frac{1}{99.100} = \frac{1}{51} + \frac{1}{52} + \frac{1}{53} + \dots + \frac{1}{100}$$

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