VINA 3 – BỒI DƯỠNG HỌC SINH GIỎI TOÁN 6
GIÁO VIÊN: NGUYỄN THÀNH LONG
PHÁT TRIỂN TỪ BÀI TOÁN DÃY SỐ QUY LUẬT QUEN THUỘC (PHẦN 2) – ĐÁP ÁN

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Bài 1: Cho $A=\frac{11}{9}+\frac{18}{16}+\frac{27}{25}+...+\frac{1766}{1764}$. Chứng minh: $40\frac{20}{43}<A<40\frac{20}{21}$

Bài giải:

Ta có: $\frac{11}{9}-1=\frac{2}{9}=\frac{2}{3^2}$

...

$\frac{1766}{1764}-1=\frac{2}{1764}=\frac{2}{42^2}$

Do đó: $A-40=\left(\frac{11}{9}-1\right)+\left(\frac{18}{16}-1\right)+\left(\frac{27}{25}-1\right)+...+\left(\frac{1766}{1764}-1\right)$

$A-40=\frac{2}{3^2}+\frac{2}{4^2}+\frac{2}{5^2}+...+\frac{2}{42^2}$

Đặt: $M=\frac{2}{3^2}+\frac{2}{4^2}+\frac{2}{5^2}+...+\frac{2}{42^2}=2.\left(\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+...+\frac{1}{42^2}\right)$

Ta có: $2.\left(\frac{1}{3.4}+\frac{1}{4.5}+\frac{1}{5.6}+...+\frac{1}{42.43}\right)<M<2.\left(\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+...+\frac{1}{41.42}\right)$

$\Rightarrow 2.\left(\frac{1}{3}-\frac{1}{43}\right)<M<2.\left(\frac{1}{2}-\frac{1}{42}\right)$

$\Rightarrow \frac{80}{129}<M<\frac{20}{21}$ Mà: $\frac{20}{43}=\frac{80}{172}<\frac{80}{129}$ nên $\frac{20}{43}<M<\frac{20}{21}$

Vậy $\frac{20}{43}<A-40<\frac{20}{21}\Rightarrow 40\frac{20}{43}<A<40\frac{20}{21}$

Bài 2: Cho $N=\frac{1.4}{2.3}+\frac{2.5}{3.4}+\frac{3.6}{4.5}+...+\frac{98.101}{99.100}$. Chứng minh: $97<N<98$

Bài giải:

Ta có: $\frac{1.4}{2.3}=\frac{(2-1).(3+1)}{2.3}=\frac{2.3-3+2-1}{2.3}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{2.3}$

$\frac{2.4}{3.4}=\frac{(3-1).(4+1)}{3.4}=\frac{3.4-4+3-1}{3.4}=1-\frac{1}{3}+\frac{1}{4}-\frac{1}{3.4}$

...

$\frac{98.101}{99.100}=\frac{(99-1)(100+1)}{99.100}=1-\frac{1}{99}+\frac{1}{100}-\frac{1}{99.100}$

Suy ra: $N=\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{2.3}\right)+\left(1-\frac{1}{3}+\frac{1}{4}-\frac{1}{3.4}\right)+...+\left(1-\frac{1}{99}+\frac{1}{100}-\frac{1}{99.100}\right)$

$N=98-\frac{1}{2}+\frac{1}{100}-\frac{1}{2.3}-\frac{1}{3.4}-...-\frac{1}{99.100}$

$N=98-\frac{1}{2}+\frac{1}{100}-\left(\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{99.100}\right)$

$N=98-\frac{1}{2}+\frac{1}{100}-\left(\frac{1}{2}-\frac{1}{100}\right)=98-\frac{1}{2}+\frac{1}{100}-\frac{1}{2}+\frac{1}{100}=98-\frac{1}{4}$

Do đó: $97<N<98$

Bài 3: Chứng minh rằng $B=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+...+\frac{100}{2^{100}}<2$

Bài giải:

$B=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+...+\frac{100}{2^{100}}$

$2B=1+\frac{2}{2}+\frac{3}{2^2}+\frac{4}{2^3}+...+\frac{100}{2^{99}}$

Suy ra: $2B-B=\left(1+\frac{2}{2}+\frac{3}{2^2}+\frac{4}{2^3}+...+\frac{100}{2^{99}}\right)-\left(\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+...+\frac{100}{2^{100}}\right)$

$B=1+\left(\frac{2}{2}-\frac{1}{2}\right)+\left(\frac{3}{2^2}-\frac{2}{2^2}\right)+\left(\frac{4}{2^3}-\frac{3}{2^3}\right)+...+\left(\frac{100}{2^{99}}-\frac{99}{2^{99}}\right)-\frac{100}{2^{100}}$

$B=1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{99}}-\frac{100}{2^{100}}$

$2B=2+1+\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^{98}}-\frac{100}{2^{99}}$

Suy ra: $2B-B=\left(2+1+\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^{98}}-\frac{100}{2^{99}}\right)-\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{99}}-\frac{100}{2^{100}}\right)$

$B = 2-\frac{100}{2^{99}}+\frac{1}{2^{99}}-\frac{100}{2^{100}} = 2-\frac{99}{2^{99}}-\frac{100}{2^{100}} < 2$

Vậy $B < 2$

Bài 4: Cho tổng: $T = \frac{2}{2^1}+\frac{3}{2^2}+\frac{4}{2^3}+...+\frac{2017}{2^{2016}}$

So sánh T với 3.

Bài giải:

$T = \frac{2}{2^1}+\frac{3}{2^2}+\frac{4}{2^3}+...+\frac{2017}{2^{2016}}$

$\frac{T}{2} = \frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+...+\frac{2017}{2^{2017}}$

$T-\frac{T}{2} = \left(\frac{2}{2^1}+\frac{3}{2^2}+\frac{4}{2^3}+...+\frac{2017}{2^{2016}}\right)-\left(\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+...+\frac{2017}{2^{2017}}\right)$

$\frac{T}{2} = \frac{2}{2^1}+\left(\frac{3}{2^2}-\frac{2}{2^2}\right)+\left(\frac{4}{2^3}-\frac{3}{2^3}\right)+...+\left(\frac{2017}{2^{2016}}-\frac{2016}{2^{2016}}\right)-\frac{2017}{2^{2017}}$

$\frac{T}{2} = 1+\left(\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{2016}}\right)-\frac{2017}{2^{2017}}$

Đặt: $M = \frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{2016}}$

$2M = \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{2015}}$

$\Rightarrow 2M - M = \left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{2015}}\right)-\left(\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{2016}}\right)$

$M = \frac{1}{2}-\frac{1}{2^{2016}} < \frac{1}{2}$

$\Rightarrow \frac{T}{2} < 1+\frac{1}{2}-\frac{2017}{2^{2017}} < 1+\frac{1}{2} = \frac{3}{2}$

Hay $T < 3$

Vậy $T < 3$.

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