**VINA 3 – BỒI DƯỠNG HỌC SINH GIỎI TOÁN 7**

**GIÁO VIÊN: NGUYỄN THÀNH LONG**

**ĐỒNG DƯ THỨC - DẠNG 2: CHỨNG MINH CHIA HẾT – ĐÁP ÁN**

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**Bài 1:** Chứng minh rằng:

a) $2^{51}-1 \vdots 7$

b) $36^{63}-1 \vdots 7$

c) $2^{2012}-4 \vdots 31$

d) $3^{105}+4^{105} \vdots 13$

e) $19^{2005}+11^{2004} \vdots 10$

f) $4^{4021}+3^{2012} \vdots 13$

g) $17^{19}+19^{17} \vdots 18$

h) $2^{70}+3^{70} \vdots 13$

***Bài giải:***

a) $2^{51}-1 \vdots 7$

Ta có: $2^3 \equiv 1 \pmod 7$

$\Rightarrow \left(2^3\right)^{17} = 2^{51} \equiv 1 \pmod 7$

Suy ra: $2^{51}-1 \equiv 0 \pmod 7$

Vậy $2^{51}-1 \vdots 7$

b) $36^{63}-1 \vdots 7$

Ta có: $36 \equiv 1 \pmod 7$

$\Rightarrow 36^{63} \equiv 1 \pmod 7$

Suy ra: $36^{63}-1 \equiv 0 \pmod 7$

Vậy $36^{63}-1 \vdots 7$

c) $\left(2^{1995}-1\right) \vdots 31$

Ta có: $2^{1995} = \left(2^5\right)^{399} = 32^{399}$

Mà: $32 \equiv 1 \pmod{31}$

$\Rightarrow 32^{399} \equiv 1^{399} \pmod{31}$

Hay $2^{1995}-1 \vdots 31$

d) $2^{2012} - 4 \vdots 31$

Ta có: $2^5 \equiv 1 \pmod{31}$

$\Rightarrow (2^5)^{402} \equiv 1 \pmod{31}$

$\Rightarrow 2^{2010}.2^2 \equiv 4 \pmod{31}$

Suy ra: $2^{2012} - 4 \equiv 0 \pmod{31}$

Vậy $2^{2012} - 4 \vdots 31$

e) $3^{105} + 4^{105} \vdots 13$

Ta có: $3^3 \equiv 1 \pmod{13}$

$\Rightarrow (3^3)^{35} \equiv 1 \pmod{13}$

Hay $3^{105} \equiv 1 \pmod{13}$

$4^2 \equiv 3 \pmod{13}$

$\Rightarrow (4^2)^3 \equiv 3^3 \equiv 1 \pmod{13}$

$\Rightarrow (4^6)^{17} = 4^{102} \equiv 1 \pmod{13}$

$\Rightarrow 4^{102}.4^3 = 4^{105} \equiv 1.4^3 \equiv 12 \pmod{13}$

Suy ra: $3^{105} + 4^{105} \equiv 1 + 12 \equiv 0 \pmod{13}$

Vậy $3^{105} + 4^{105} \vdots 13$

f) $19^{2005} + 11^{2004} \vdots 10$

Ta có: $19^2 = 361 \equiv 1 \pmod{10}$

$\Rightarrow (19^2)^{1002} = 19^{2004} \equiv 1 \pmod{10}$

$\Rightarrow 19^{2004}.19 = 19^{2005} \equiv 1.19 \equiv 9 \pmod{10}$

$11 \equiv 1 \pmod{10}$

$\Rightarrow 11^{2004} \equiv 1 \pmod{10}$

Suy ra: $19^{2005} + 11^{2004} \equiv 9 + 1 \equiv 0 \pmod{10}$

Vậy $19^{2005} + 11^{2004} \vdots 10$

g) $4^{4021}+3^{2012} \vdots 13$

Ta có:

$$4^{4021}+3^{2012}=4.4^{4020}+3^2.3^{2010}=4.\left(4^2\right)^{2010}+3^2.3^{2010}=4.16^{2010}+9.3^{2010}=\left(4.16^{2010}-4.3^{2010}\right)+13.3^{2010}$$

Mà: $16^{2010}-3^{2010} \vdots (16-3)$

$4.\left(16^{2010}-3^{2010}\right)\equiv 0 \pmod{13}$

$\Rightarrow 4.\left(16^{2010}-3^{2010}\right)+13.3^{2010} \equiv 0 \pmod{13}$

Vậy $4^{4021}+3^{2012} \vdots 13$

h) $17^{19}+19^{17} \vdots 18$

Ta có: $17^2=289\equiv 1 \pmod{18}$

$\Rightarrow \left(17^2\right)^9=17^{18}\equiv 1 \pmod{18}$

$\Rightarrow 17^{18}.17=17^{19} \equiv 1.\ 17 \equiv 17 \pmod{18}$

$19 \equiv 1 \pmod{18}$

$\Rightarrow 19^{17}\equiv 1 \pmod{18}$

Suy ra: $17^{19}+19^{17} \equiv 1+17 \equiv 0 \pmod{18}$

Vậy $17^{19}+19^{17} \vdots 18$

f) $2^{70}+3^{70} \vdots 13$

Ta có: $2^4\equiv 3 \pmod{13}$

$\Rightarrow \left(2^4\right)^3=2^{12}\equiv 3^3\equiv 1 \pmod{13}$

$\Rightarrow \left(2^{12}\right)^5=2^{60}\equiv 1 \pmod{13}$

Và $2^{10}\equiv 10 \pmod{13}$

Suy ra: $2^{70}\equiv 1.10\equiv 10 \pmod{13}$

$3^3\equiv 1 \pmod{13}$

$\Rightarrow \left(3^3\right)^{23}=3^{69}\equiv 1 \pmod{13}$

$\Rightarrow 3^{69}.3 \equiv 1.3 \pmod{13}$

Suy ra: $2^{70} + 3^{70} \equiv 10 + 3 \equiv 0 \pmod{13}$

Vậy $2^{70} + 3^{70} \vdots 13$

**Bài 2:** Cho n là số tự nhiên. Chứng minh rằng:

a) $2^{4n} - 1 \vdots 15$

b) $8^n + 6 \vdots 7$

c) $7.5^{2n} + 12.6^n \vdots 19$

d) $6^{2n+1} + 5^{n+2} \vdots 31$

e) $5^{n+2} + 26.5^n + 8^{2n+1} \vdots 59$

f) $3^{n+2} + 4^{2n+1} \vdots 13$

g) $6^{2n} + 19^n - 2^{n+1} \vdots 17$

h) $11^{n+2} + 12^{2n+1} \vdots 133$

***Bài giải:***

a) $2^{4n} - 1 \vdots 15$

Ta có: $2^4 \equiv 1 \pmod{15}$

$\Rightarrow (2^4)^n = 2^{4n} \equiv 1 \pmod{15}$

Suy ra: $2^{4n} - 1 \equiv 0 \pmod{15}$

Vậy $2^{4n} - 1 \vdots 15$

b) $8^n + 6 \vdots 7$

Ta có: $8^n - 1^n \vdots (8-1)$

Do đó: $8^n \equiv 1 \pmod{7}$

$\Rightarrow 8^n + 6 \equiv 1 + 6 \equiv 0 \pmod{7}$

c) $7.5^{2n} + 12.6^n \vdots 19$

Ta có: $5^2 = 25 \equiv 6 \pmod{19}$

$\Rightarrow (5^2)^n \equiv 6^n \pmod{19}$

$\Rightarrow 7.5^{2n} \equiv 7.6^n \pmod{7.\ 19}$

Suy ra: $7.5^{2n} + 12.6^n \equiv 7.6^n + 12.6^n \equiv 19.6^n \equiv 0 \pmod{7.19}$

Vậy $7.5^{2n} + 12.6^n \vdots 19$

b) $6^{2n+1}+5^{n+2}\ \vdots\ 31$

Ta có: $6^2\equiv 5\ (\text{mod } 31)$

$\Rightarrow (6^2)^n\equiv 5^n\ (\text{mod } 31)$

$\Rightarrow 6^{2n+1}\equiv 6.5^{n+1}\ (\text{mod } 31)$

Suy ra: $6^{2n+1}+5^{n+2}\equiv 6.5^n+5^{n+2}\equiv 5^n.(6+5^2)\equiv 5^n.31\equiv 0\ (\text{mod } 31)$

Vậy $6^{2n+1}+5^{n+2}\ \vdots\ 31$

d) $5^{n+2}+26.5^n+8^{2n+1}\ \vdots\ 59$

Ta có: $8^2=64\equiv 5\ (\text{mod } 59)$

$\Rightarrow (8^2)^n\equiv 5^n\ (\text{mod } 59)$

$\Rightarrow 8^{2n}.8=8^{2n+1}\equiv 5^n.8\ (\text{mod } 59)$

Suy ra: $5^{n+2}+26.5^n+8^{2n+1}\equiv 5^n.5^2+26.5^n+8.5^n=5^n.(5^2+26+8)=5^n.59\equiv 0\ (\text{mod } 59)$

Vậy $5^{n+2}+26.5^n+8^{2n+1}\ \vdots\ 59$

f) $3^{n+2}+4^{2n+1}\ \vdots\ 13$

Ta có: $4^2\equiv 3\ (\text{mod } 13)$

$\Rightarrow 4^{2n}\equiv 3^n\ (\text{mod } 13)$

$\Rightarrow 4^{2n}.4=4^{2n+1}\equiv 3^n.4\ (\text{mod } 13)$

Suy ra: $3^{n+2}+4^{2n+1}\equiv 3^n.3^2+3^n.4=3^n.(3^2+4)=3^n.13\equiv 0\ (\text{mod } 13)$

Vậy $3^{n+2}+4^{2n+1}\ \vdots\ 13$

g) $6^{2n}+19^n-2^{n+1}\ \vdots\ 17$

Ta có: $6^2\equiv 2\ (\text{mod } 17)$

$\Rightarrow (6^2)^n=6^{2n}\equiv 2^n\ (\text{mod } 17)$

$19\equiv 2\ (\text{mod } 17)$

$\Rightarrow 19^n\equiv 2^n\ (\text{mod } 17)$

Suy ra: $6^{2n}+19^n-2^{n+1}\equiv 2^n+2^n-2^{n+1}=2.2^n-2^{n+1}\equiv 0$ (mod 17)

Vậy $6^{2n}+19^n-2^{n+1}\vdots 17$

h) $11^{n+2}+12^{2n+1}\ \vdots\ 133$

Ta có: $11^{n+2}+12^{2n+1}=11^n.11^2+12.(12^2)^n=(133-12).11^n+12.144^n=133.11^n-12.11^n+12.144^n$

$=133.11^n+12.(144^n-11^n)$

Mà: $144\equiv 11$ (mod 133)

$\Rightarrow 144^n\equiv 11^n$ (mod 133)

Suy ra: $144^n-11^n\equiv 0$ (mod 133)

$\Rightarrow 133.11^n+12.(144^n-11^n)\equiv 0$ (mod 133)

Vậy $11^{n+2}+12^{2n+1}\ \vdots\ 133$

Bài 3: Cho số tự nhiên n. Chứng minh rằng:

a) $5^{2n+1}+2^{n+4}+2^{n+1}\ \vdots 23$

b) $13^{n+2}+14^{2n+1}\ \vdots\ 183$

c) $5^{2n+1}.2^{n+2}+3^{n+2}.2^{2n+1}\ \vdots\ 38$

***Bài giải:***

a) $5^{2n+1}+2^{n+4}+2^{n+1}$

Ta có: $5^2\equiv 2$ (mod 23)

$\Rightarrow (5^2)^n.5=5^{2n+1}\equiv 2^n.5$ (mod 23)

Khi đó ta được: $5^{2n+1}+2^{n+4}+2^{n+1}\equiv 5.2^n+16.2^n+2.2^n=23.2^n$ chia hết cho 23.

b) $13^{n+2}+14^{2n+1}\ \vdots\ 183$

Ta có: $14^2\equiv 13$ (mod 183)

$\Rightarrow (14^2)^n.14=14^{2n+1}\equiv 13^n.14$

Khi đó ta được: $13^{n+2}+14^{2n+1}\equiv 13^n.13^2+13^n.14=13^n.(13^2+14)=13^n.183\equiv 0$ (mod 183)

Vậy $13^{n+2}+14^{2n+1}\ \vdots\ 183$

c) $5^{2n+1}.2^{n+2}+3^{n+2}.2^{2n+1}\ \vdots\ 38$

Ta có: $5^{2n+1}.2^{n+2}+3^{n+2}.2^{2n+1}=5^{2n}.5.2^{n}.2^{2}+3^{n}.3^{2}.2^{2n}.2$

$=20.50^{n}+18.12^{n}$

Ta có: $50\equiv 12\ (\text{mod }38)$

$\Rightarrow 50^{n}\equiv 12^{n}$

Khi đó ta được: $20.50^{n}+18.12^{n}\equiv 20.12^{n}+18.12^{n}=38.12^{n}\equiv 0\,(\text{mod }38)$

Vậy $5^{2n+1}.2^{n+2}+3^{n+2}.2^{2n+1}\ \vdots\ 38$

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