

VINA 3 – BỒI DƯỠNG HỌC SINH GIỎI TOÁN 6
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MỘT SỐ BÀI TOÁN VỀ DÃY PHÂN SỐ CÓ QUY LUẬT – ĐÁP ÁN

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Bài 1: Tính tổng sau:

$$a) S = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^{100}}$$

$$b) B = 512 - \frac{512}{2} - \frac{512}{2^2} - \frac{512}{2^3} - \dots - \frac{512}{2^{10}}$$

Bài giải:

$$a) S = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^{100}}$$

$$3S = 3 + 1 + \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^{99}}$$

$$3S - S = \left(3 + 1 + \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^{99}} \right) - \left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^{100}} \right)$$

$$2S = 3 - \frac{1}{3^{100}} = \frac{3^{100} - 1}{3^{100}}$$

$$S = \frac{3^{99} - 1}{2 \cdot 3^{100}}$$

$$b) \text{Đặt } N = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{10}}$$

$$2N = 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^9}$$

$$2N - N = \left(1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^9} \right) - \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{10}} \right)$$

$$N = 1 - \frac{1}{2^{10}}$$

$$\text{Suy ra: } B = 512 - 512 \cdot \left(1 - \frac{1}{2^{10}} \right) = 512 - 512 + 512 \cdot \frac{1}{2^{10}} = \frac{1}{2}$$

Bài 2: Tính các tổng sau bằng phương pháp hợp lý nhất:

$$a) A = \frac{4}{3.7} + \frac{4}{7.11} + \frac{4}{11.15} + \dots + \frac{4}{107.111}$$

$$b) B = \frac{6}{15.18} + \frac{6}{18.21} + \frac{6}{21.24} + \dots + \frac{6}{87.90}$$

Bài giải:

$$a) A = \frac{4}{3.7} + \frac{4}{7.11} + \frac{4}{11.15} + \dots + \frac{4}{107.111}$$

$$A = \frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{11} + \frac{1}{11} - \frac{1}{15} + \dots + \frac{1}{107} - \frac{1}{111}$$

$$A = \frac{1}{3} - \frac{1}{111} = \frac{12}{37}$$

$$b) B = \frac{6}{15.18} + \frac{6}{18.21} + \frac{6}{21.24} + \dots + \frac{6}{87.90}$$

$$B = 2 \cdot \left(\frac{3}{15.18} + \frac{3}{18.21} + \frac{3}{21.24} + \dots + \frac{3}{87.90} \right)$$

$$B = 2 \cdot \left(\frac{1}{15} - \frac{1}{18} + \frac{1}{18} - \frac{1}{21} + \frac{1}{21} - \frac{1}{24} + \dots + \frac{1}{87} - \frac{1}{90} \right)$$

$$B = 2 \cdot \left(\frac{1}{15} - \frac{1}{90} \right) = 2 \cdot \frac{1}{18} = \frac{1}{9}$$

Bài 3: Tìm $x \in \mathbb{N}$, biết:

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \dots + \frac{2}{x(x+1)} = \frac{2016}{2018}$$

Bài giải:

$$\frac{2}{6} + \frac{2}{12} + \frac{2}{20} + \dots + \frac{2}{x(x+1)} = \frac{2016}{2018}$$

$$2 \cdot \left(\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{x(x+1)} \right) = \frac{2016}{2018}$$

$$2 \cdot \left(\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{x(x+1)} \right) = \frac{2016}{2018}$$

$$2 \cdot \left(\frac{1}{2} - \frac{1}{x+1} \right) = \frac{2016}{2018}$$

$$\frac{1}{2} - \frac{1}{x+1} = \frac{504}{1009}$$

$$\frac{1}{x+1} = \frac{1}{2018}$$

$$x+1=2018$$

$$x=2017$$

Bài 4: Tính tổng:

$$a) A = \frac{2}{1.2.3} + \frac{2}{2.3.4} + \frac{2}{3.4.5} + \dots + \frac{2}{23.24.25}$$

$$b) B = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{998.999.1000}$$

Bài giải:

$$a) A = \frac{2}{1.2.3} + \frac{2}{2.3.4} + \frac{2}{3.4.5} + \dots + \frac{2}{23.24.25}$$

$$A = \frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \dots + \frac{1}{23.24} - \frac{1}{24.25}$$

$$A = \frac{1}{1.2} - \frac{1}{24.25} = \frac{299}{600}$$

$$b) B = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{998.999.1000}$$

$$2B = \frac{2}{1.2.3} + \frac{2}{2.3.4} + \frac{2}{3.4.5} + \dots + \frac{2}{998.999.1000}$$

$$2B = \frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \dots + \frac{1}{998.999} - \frac{1}{999.1000}$$

$$2B = \frac{1}{1.2} - \frac{1}{999.1000} = \frac{499499}{999000}$$

$$B = \frac{499499}{1998000}$$

Bài 5: Thực hiện tính: $E = 1 + \frac{1}{2} \cdot (1+2) + \frac{1}{3} \cdot (1+2+3) + \frac{1}{4} \cdot (1+2+3+4) + \dots + \frac{1}{200} \cdot (1+2+\dots+200)$

Bài giải:

$$E = 1 + \frac{1}{2} \cdot (1+2) + \frac{1}{3} \cdot (1+2+3) + \frac{1}{4} \cdot (1+2+3+4) + \dots + \frac{1}{200} \cdot (1+2+\dots+200)$$

$$E = 1 + \frac{1}{2} \cdot \frac{2.3}{2} + \frac{1}{3} \cdot \frac{3.4}{2} + \frac{1}{4} \cdot \frac{4.5}{2} + \dots + \frac{1}{200} \cdot \frac{200.201}{2}$$

$$E = 1 + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \dots + \frac{201}{2}$$

$$E + \frac{1}{2} = \frac{1+2+3+\dots+201}{2}$$

$$E = \frac{1}{2} \cdot \frac{201 \cdot 202}{2} - \frac{1}{2} = 10150$$

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