BỒI DƯỠNG NĂNG LỰC TOÁN 9

ĐỀ BÀI TẬP VỀ NHÀ NGÀY 02.07

Giáo viên: Trần Ngọc Hà

Tài liệu lớp học trực tiếp 9A0.1– 18h – 21h – Tối thứ 6 – 23/26 Nguyên Hồng

Họ và tên: ....Ngày học: ....

ĐẠI SỐ

Câu 18. Cho biểu thức $S_n=\left(2-\sqrt{3}\right)^n+\left(2+\sqrt{3}\right)^n$ (với n nguyên dương)

a) Chứng minh rằng $S_{3n}+3S_n=S_n^3$

b) Tính S_3,S_9

Hướng dẫn giải

a) $S_{2n}=\left(\sqrt{3}+\sqrt{2}\right)^{2n}+\left(\sqrt{3}-\sqrt{2}\right)^{2n}$

$$=\left[\left(\sqrt{3}+\sqrt{2}\right)^n\right]^2+2.\left(\sqrt{3}+\sqrt{2}\right)^n.\left(\sqrt{3}-\sqrt{2}\right)^n+\left[\left(\sqrt{3}-\sqrt{2}\right)^n\right]^2-2$$

$$=\left[\left(\sqrt{3}+\sqrt{2}\right)^n+\left(\sqrt{3}-\sqrt{2}\right)^n\right]^2-2$$

$$=S_n^2-2(dpcm)$$

b) $S_3=\left(2-\sqrt{3}\right)^3+\left(2+\sqrt{3}\right)^3=8-12\sqrt{3}+18-9\sqrt{3}+8+12\sqrt{3}+18+9\sqrt{3}=52$

$S_9=\left(2-\sqrt{3}\right)^9+\left(2+\sqrt{3}\right)^9=\left[\left(2-\sqrt{3}\right)^3\right]^3+\left[\left(2+\sqrt{3}\right)^3\right]^3$

$S_9=\left[\left(2-\sqrt{3}\right)^3+\left(2+\sqrt{3}\right)^3\right]\left[\left(2-\sqrt{3}\right)^6-\left(2-\sqrt{3}\right)^3\left(2+\sqrt{3}\right)^3+\left(2+\sqrt{3}\right)^6\right]$

$S_9=52\left(S_{2\times3}-1\right)=52\left(S_3^2-3\right)=52\left(52^2-3\right)=140452$

Câu 19. Chứng minh đẳng thức

a) $\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+4}=\sqrt{2}-1$

b) $\sqrt{21-6\sqrt{6}}+\sqrt{9+2\sqrt{18}}-2\sqrt{6+3\sqrt{3}}=0$

c) $\sqrt{6+2\sqrt{5-\sqrt{13+\sqrt{48}}}}=1+\sqrt{3}$

d) $\sqrt{4+\sqrt{5\sqrt{3}+5\sqrt{48-10\sqrt{7+4\sqrt{3}}}}}=3$

Hướng dẫn giải:

a) $VT=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+4}=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{\left(\sqrt{2}+\sqrt{3}+2\right)+\left(\sqrt{6}+\sqrt{8}+2\right)}$

$$=\frac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)+\sqrt{2}\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)}=\frac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)\left(\sqrt{2}+1\right)}=\frac{1}{\sqrt{2}+1}=\sqrt{2}-1=VP$$

b) $VT=\sqrt{21-6\sqrt{6}}+\sqrt{9+2\sqrt{18}}-2\sqrt{6+3\sqrt{3}}$

$$=\sqrt{18-2.3\sqrt{2}.\sqrt{3}+3}+\sqrt{6+2.\sqrt{3}.\sqrt{6}+3}-\sqrt{2}.\sqrt{12+6\sqrt{3}}$$

$$=\sqrt{\left(3\sqrt{2}-\sqrt{3}\right)^2}+\sqrt{\left(\sqrt{6}+\sqrt{3}\right)^2}-\sqrt{2}.\sqrt{9+2.3.\sqrt{3}+3}$$

$$=3\sqrt{2}-\sqrt{3}+\sqrt{6}+\sqrt{3}-\sqrt{2}.\sqrt{\left(3+\sqrt{3}\right)^2}$$

$$=3\sqrt{2}-\sqrt{3}+\sqrt{6}+\sqrt{3}-\sqrt{2}\left(3+\sqrt{3}\right)=3\sqrt{2}-\sqrt{3}+\sqrt{6}+\sqrt{3}-3\sqrt{2}-\sqrt{6}=0=VP$$

c) $VT=\sqrt{6+2\sqrt{5-\sqrt{13+\sqrt{48}}}}=\sqrt{6+2\sqrt{5-\sqrt{13+4\sqrt{3}}}}=\sqrt{6+2\sqrt{5-\sqrt{\left(2\sqrt{3}+1\right)^2}}}$

$$=\sqrt{6+2\sqrt{4-2\sqrt{3}}}=\sqrt{6+2\sqrt{\left(\sqrt{3}-1\right)^2}}=\sqrt{4+2\sqrt{3}}=\sqrt{\left(\sqrt{3}+1\right)^2}=\sqrt{3}+1=VP$$

d) $VT=\sqrt{4+\sqrt{5\sqrt{3}+5\sqrt{48-10\sqrt{7+4\sqrt{3}}}}}=\sqrt{4+\sqrt{5\sqrt{3}+5\sqrt{48-10\sqrt{\left(2+\sqrt{3}\right)^2}}}}$

$$=\sqrt{4+\sqrt{5\sqrt{3}+5\sqrt{28-10\sqrt{3}}}}=\sqrt{4+\sqrt{5\sqrt{3}+5\sqrt{\left(5-\sqrt{3}\right)^2}}}$$

$$=\sqrt{4+\sqrt{25}}=\sqrt{9}=3=VP$$

Câu 20. Rút gọn các BT sau

a) $A=\dfrac{2\sqrt{15}-2\sqrt{10}+\sqrt{6}-3}{2\sqrt{5}-2\sqrt{10}-\sqrt{3}+\sqrt{6}}$

b) $B=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+\sqrt{16}}{\sqrt{2}+\sqrt{3}+\sqrt{4}}$

c) $C=\left(\sqrt{\dfrac{2}{3}}+\sqrt{\dfrac{3}{2}}+2\right)\left(\dfrac{\sqrt{2}+\sqrt{3}}{4\sqrt{2}}+\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)\left(24+8\sqrt{6}\right)$

Hướng dẫn giải:

a) $A=\dfrac{2\sqrt{15}-2\sqrt{10}+\sqrt{6}-3}{2\sqrt{5}-2\sqrt{10}-\sqrt{3}+\sqrt{6}}=\dfrac{\sqrt{3}\left(2\sqrt{5}-\sqrt{3}\right)-\sqrt{2}\left(2\sqrt{5}-\sqrt{3}\right)}{\left(2\sqrt{5}-\sqrt{3}\right)-\sqrt{2}\left(2\sqrt{5}-\sqrt{3}\right)}$

$$=\frac{\left(2\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}-\sqrt{2}\right)}{\left(2\sqrt{5}-\sqrt{3}\right)\left(1-\sqrt{2}\right)}=\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-1}=\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}+1\right)$$

b) $B=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+\sqrt{16}}{\sqrt{2}+\sqrt{3}+\sqrt{4}}=\dfrac{\left(\sqrt{2}+\sqrt{3}+2\right)+\left(\sqrt{6}+\sqrt{8}+2\right)}{\sqrt{2}+\sqrt{3}+\sqrt{4}}$

$$=\frac{\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)+\sqrt{2}\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)}{\sqrt{2}+\sqrt{3}+\sqrt{4}}=\frac{\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)\left(\sqrt{2}+1\right)}{\sqrt{2}+\sqrt{3}+\sqrt{4}}=\sqrt{2}+1$$

c) $C=\left(\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{2}}+2\right)\left(\frac{\sqrt{2}+\sqrt{3}}{4\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)\left(24+8\sqrt{6}\right)$

$$=\left(\frac{5}{\sqrt{6}}+2\right)\left[\frac{\left(\sqrt{2}+\sqrt{3}\right)^2+4\sqrt{6}}{4\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)}\right].\sqrt{6}.4\sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)=\left(5+2\sqrt{6}\right)\left(5+6\sqrt{6}\right)=97+40\sqrt{6}$$

Câu 21. So sánh các số sau

a) $\sqrt{21}+\sqrt{2}$ và $\sqrt{14}+\sqrt{3}$

b) $\sqrt{17}+\sqrt{6}$ và $\sqrt{21}-\sqrt{2}$

Hướng dẫn giải:

a) Ta có: $\left(\sqrt{21}+\sqrt{2}\right)^2=23+2\sqrt{42}$

$\left(\sqrt{14}+\sqrt{3}\right)^2=17+2\sqrt{42}$

Vì $23+2\sqrt{42}>17+2\sqrt{42}\Rightarrow\left(\sqrt{21}+\sqrt{2}\right)^2>\left(\sqrt{14}+\sqrt{3}\right)^2$

$\Leftrightarrow\sqrt{21}+\sqrt{2}>\sqrt{14}+\sqrt{3}$

b) Ta có: $\left(\sqrt{17}+\sqrt{6}\right)^2=23+2\sqrt{17}.\sqrt{6}$

$\left(\sqrt{21}-\sqrt{2}\right)^2=23-2.\sqrt{21}.\sqrt{2}$

Vì $23+2\sqrt{17}.\sqrt{6}>23-2.\sqrt{21}.\sqrt{2}\Rightarrow\left(\sqrt{17}+\sqrt{6}\right)^2>\left(\sqrt{21}-\sqrt{2}\right)^2$

$\Leftrightarrow\sqrt{17}+\sqrt{6}>\sqrt{21}-\sqrt{2}$

Câu 22. So sánh hai số sau $\sqrt{29}-\sqrt{28}$ và $\sqrt{28}-\sqrt{27}$.

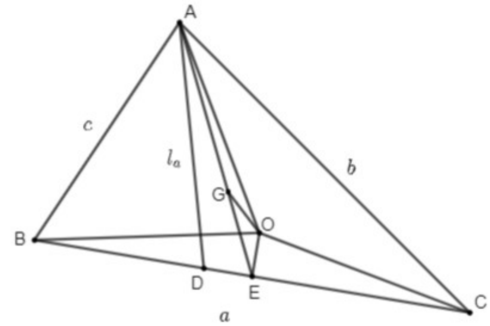
Hướng dẫn giải:

$$\frac{1}{\sqrt{29}+\sqrt{28}}<\frac{1}{\sqrt{28}+\sqrt{27}}$$

$$\Leftrightarrow\frac{\sqrt{29}-\sqrt{28}}{\left(\sqrt{29}-\sqrt{28}\right)\left(\sqrt{29}+\sqrt{28}\right)}<\frac{\sqrt{28}-\sqrt{27}}{\left(\sqrt{28}-\sqrt{27}\right)\left(\sqrt{28}+\sqrt{27}\right)}$$

$$\Leftrightarrow\sqrt{29}-\sqrt{28}<\sqrt{28}-\sqrt{27}$$

HÌNH HỌC



Câu 7. Chứng minh các đẳng thức trong tam giác:

[5] $l_a = \dfrac{2bc\cos\dfrac{A}{2}}{b+c}$

[6] $3\left(OA^2+OB^2+OC^2\right)=9OG^2+a^2+b^2+c^2$

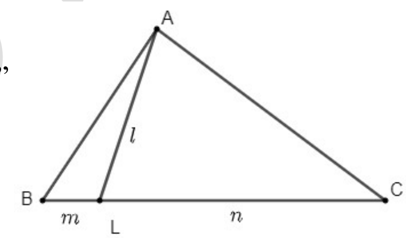
Hướng dẫn giải:

[5] Ta có: $S_{\vartriangle ABC}=S_{\vartriangle ABD}+S_{\vartriangle ADC} \Leftrightarrow \dfrac{1}{2}bc\sin A=\dfrac{1}{2}l_a c\sin\dfrac{A}{2}+\dfrac{1}{2}l_a b\sin\dfrac{A}{2}$

$\Leftrightarrow bc\sin A=l_a c\sin\dfrac{A}{2}+l_a b\sin\dfrac{A}{2} \Leftrightarrow 2bc\sin\dfrac{A}{2}\cos\dfrac{A}{2}=l_a c\sin\dfrac{A}{2}+l_a b\sin\dfrac{A}{2}$

$\Leftrightarrow 2bc\cos\dfrac{A}{2}=l_a\left(c+b\right) \Leftrightarrow l_a=\dfrac{2bc\cos\dfrac{A}{2}}{b+c}\left(dpcm\right)$

[6] Định lý Stewart: "Nếu L là điểm nằm trên cạnh BC của $\vartriangle ABC$ và nếu $AL=l; BL=m; LC=n$ thì $a\left(l^2+mn\right)=b^2m+c^2n$"



Chứng minh: Gọi α là góc giữa CL và AL;

α' là góc bù của α và $\cos\alpha=-\cos\alpha'$

Xét $\vartriangle ALC$ có: $b^2=l^2+n^2-2l.n.\cos\alpha$

Xét $\vartriangle ABL$ có: $c^2=l^2+m^2-2l.m.\cos\alpha'=l^2+m^2+2l.m.\cos\alpha$

$\Rightarrow b^2m+c^2n=l^2m+mn^2+l^2n+m^2n=l^2\left(m+n\right)+mn\left(m+n\right)$

$=\left(m+n\right)\left(l^2+mn\right)=a\left(l^2+mn\right)$

Gọi E là trung điểm của BC.

Áp dụng định lý Stewart cho $\vartriangle OAE$ có:

$AE\left(OG^2+AG.GE\right)=EO^2.AG+AO^2.GE$

Mặt khác $AO=R; AG=\dfrac{2}{3}AE; GE=\dfrac{1}{3}AE$ nên $OG^2+\dfrac{2}{9}AE^2=\dfrac{2}{3}EO^2+\dfrac{1}{3}R^2$

Vì $EA^2=\dfrac{2\left(b^2+c^2\right)-a^2}{4}$ (công thức tính độ dài đường trung tuyến) và $EO^2=R^2-\dfrac{a^2}{4}$ nên:

$OG^2=\left(R^2-\dfrac{a^2}{4}\right).\dfrac{2}{3}+\dfrac{1}{3}R^2-\dfrac{2}{9}.\dfrac{2\left(b^2+c^2\right)-a^2}{4}$

$=R^2-\dfrac{a^2}{6}-\dfrac{2\left(b^2+c^2\right)-a^2}{18}=R^2-\dfrac{a^2+b^2+c^2}{9}$

$\Leftrightarrow 3\left(OA^2+OB^2+OC^2\right)=9OG^2+a^2+b^2+c^2\left(dpcm\right)$

Câu 8. Chứng minh các BĐT trong tam giác:

[3] $ab\left(a+b\right)+bc\left(b+c\right)+ca\left(c+a\right)>3abc$

[5] $\dfrac{a^2}{p-a}+\dfrac{b^2}{p-b}+\dfrac{c^2}{p-c}\geq 2\left(a+b+c\right)$

[6] $\dfrac{a}{p-a}+\dfrac{b}{p-b}+\dfrac{c}{p-c}\geq\dfrac{3}{2}$

[9] $\dfrac{a}{b+c}+\dfrac{b}{c+a}+\dfrac{c}{b+a}+\dfrac{3abc}{\left(a+b\right)\left(b+c\right)\left(c+a\right)}<2$

Hướng dẫn giải:

[3] $ab\left(a+b\right)+bc\left(b+c\right)+ca\left(c+a\right)>3abc$

$\Leftrightarrow ab\left(a+b\right)-abc+bc\left(b+c\right)-abc+ca\left(c+a\right)-abc>0$

$\Leftrightarrow ab\left(a+b-c\right)+bc\left(b+c-a\right)+ca\left(c+a-b\right)>0 \qquad (1)$

Vì a,b,c là độ dài ba cạnh của tam giác nên

$\begin{cases}a+b-c>0\\ b+c-a>0\\ c+a-b>0\\ a,b,c>0\end{cases}\Rightarrow(1)$ luôn đúng $\Rightarrow dpcm$

[5] $\dfrac{a^2}{p-a}+\dfrac{b^2}{p-b}+\dfrac{c^2}{p-c}\geq 2\left(a+b+c\right)$

Áp dụng bất đẳng thức Cauchy – Schwarz dạng Engel ta có:

$\dfrac{a^2}{p-a}+\dfrac{b^2}{p-b}+\dfrac{c^2}{p-c}\geq\dfrac{\left(a+b+c\right)^2}{p-a+p-b+p-c}=2\left(a+b+c\right)\left(dpcm\right)2$

[6] $\dfrac{a}{p-a}+\dfrac{b}{p-b}+\dfrac{c}{p-c}\geq\dfrac{3}{2}$

$\Leftrightarrow\dfrac{a}{p-a}+1+\dfrac{b}{p-b}+1+\dfrac{c}{p-c}+1-3\geq\dfrac{3}{2}$

$\Leftrightarrow\dfrac{p}{p-a}+\dfrac{p}{p-b}+\dfrac{p}{p-c}-3\geq\dfrac{3}{2}\Leftrightarrow\dfrac{a+b+c}{b+c}+\dfrac{a+b+c}{c+a}+\dfrac{a+b+c}{a+b}-3\geq\dfrac{3}{2}$

$\Leftrightarrow\left(a+b+c\right)\left(\dfrac{1}{a+b}+\dfrac{1}{b+c}+\dfrac{1}{c+a}\right)\geq\dfrac{9}{2} \qquad (2)$

Áp dụng bất đẳng thức Cauchy cho ba số dương $a+b;b+c;c+a$ ta có:

$$\left[(a+b)+(b+c)+(c+a)\right]\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right)\geq 3\sqrt[3]{(a+b)(b+c)(c+a)}.3\sqrt[3]{\frac{1}{(a+b)(b+c)(c+a)}}=9$$

(Dấu "=" xảy ra $\Leftrightarrow a=b=c$)

$$\Leftrightarrow \frac{(a+b)+(b+c)+(c+a)}{3}\geq \frac{3}{\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}}$$

$$\Leftrightarrow 2(a+b+c)\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right)\geq 9$$

Suy ra (2) luôn đúng $\Rightarrow dpcm$

[9] $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{b+a}+\frac{3abc}{(a+b)(b+c)(c+a)}<2$

$$\Leftrightarrow a(c+a)(b+a)+b(b+c)(b+a)+c(b+c)(c+a)+3abc<2(a+b)(b+c)(c+a)$$

$$\Leftrightarrow a^3+a^2b+2abc+a^2c+b^3+ab^2+b^2c+abc+bc^2+abc+c^3+ac^2<$$

$$<2\left(abc+a^2b+ac^2+a^2c+b^2c+ab^2+bc^2+abc\right)$$

$\Leftrightarrow (a+b-c)(b+c-a)(c+a-b)>0$ (luôn đúng) $\Rightarrow dpcm$