

BỒI DƯỠNG HỌC SINH GIỎI VÀ THI CHUYÊN
HƯỚNG DẪN BÀI TẬP VỀ NHÀ
Tài liệu lớp học Zoom 7M1 - 18h00 - 21h15 - Tối thứ 2

Họ và tên:Ngày học:

CA 1

Câu 11. So sánh F và G biết:

$$F = 124 \left(\frac{1}{1 \times 1985} + \frac{1}{2 \times 1986} + \dots + \frac{1}{16 \times 2000} \right)$$

$$G = \frac{1}{1 \times 17} + \frac{1}{2 \times 18} + \dots + \frac{1}{1984 \times 2000}$$

HD:

$$F = 124 \left(\frac{1}{1 \times 1985} + \frac{1}{2 \times 1986} + \dots + \frac{1}{16 \times 2000} \right)$$

$$F = \frac{124}{1984} \times \left(\frac{1}{1} - \frac{1}{1985} + \frac{1}{2} - \frac{1}{1986} + \dots + \frac{1}{16} - \frac{1}{2000} \right)$$

$$F = \frac{1}{16} \left[\left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{16} \right) - \left(\frac{1}{1985} + \dots + \frac{1}{2000} \right) \right]$$

$$G = \frac{1}{1 \times 17} + \frac{1}{2 \times 18} + \dots + \frac{1}{1984 \times 2000}$$

$$G = \frac{1}{16} \left(\frac{1}{1} - \frac{1}{17} + \frac{1}{2} - \frac{1}{18} + \dots + \frac{1}{1984} - \frac{1}{2000} \right)$$

$$G = \frac{1}{16} \left[\left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{1984} \right) - \left(\frac{1}{17} + \frac{1}{18} + \dots + \frac{1}{2000} \right) \right]$$

$$G = \frac{1}{16} \left[\left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{16} \right) + \left(\frac{1}{17} + \frac{1}{18} + \dots + \frac{1}{1984} \right) - \left(\frac{1}{17} + \frac{1}{18} + \dots + \frac{1}{1984} \right) - \left(\frac{1}{1985} + \dots + \frac{1}{2000} \right) \right]$$

$$G = \frac{1}{16} \left[\left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{16} \right) - \left(\frac{1}{1985} + \dots + \frac{1}{2000} \right) \right]. \text{ Vậy } F = G$$

Câu 12. So sánh hai biểu thức A và B:

$$A = 124 \left(\frac{1}{1.1985} + \frac{1}{2.1986} + \frac{1}{3.1987} + \dots + \frac{1}{16.2000} \right)$$

$$B = \frac{1}{1.17} + \frac{1}{2.18} + \frac{1}{3.19} + \dots + \frac{1}{1984.2000}$$

HD:

$$\begin{aligned}
 A &= 124 \cdot \left(\frac{1}{1.1985} + \frac{1}{2.1986} + \frac{1}{3.1987} + \dots + \frac{1}{16.2000} \right) \\
 &= \frac{1}{16} \left(\frac{1984}{1.1985} + \frac{1984}{2.1986} + \frac{1984}{3.1987} + \dots + \frac{1984}{16.2000} \right) \\
 &= \frac{1}{16} \left(1 - \frac{1}{1985} + \frac{1}{2} - \frac{1}{1986} + \frac{1}{3} - \frac{1}{1987} + \dots + \frac{1}{16} - \frac{1}{2000} \right) \\
 &= \frac{1}{16} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{16} - \frac{1}{1985} - \frac{1}{1986} - \frac{1}{1987} - \dots - \frac{1}{2000} \right)
 \end{aligned}$$

$$\begin{aligned}
 B &= \frac{1}{1.17} + \frac{1}{2.18} + \frac{1}{3.19} + \dots + \frac{1}{1984.2000} \\
 &= \frac{1}{16} \left(\frac{16}{1.17} + \frac{16}{2.18} + \frac{16}{3.19} + \dots + \frac{16}{1984.2000} \right) \\
 &= \frac{1}{16} \left(1 - \frac{1}{17} + \frac{1}{2} - \frac{1}{18} + \frac{1}{3} - \frac{1}{19} + \dots + \frac{1}{1984} - \frac{1}{2000} \right) \\
 &= \frac{1}{16} \left[\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{16} \right) + \left(\frac{1}{17} + \frac{1}{18} + \dots + \frac{1}{1984} \right) - \left(\frac{1}{17} + \frac{1}{18} + \dots + \frac{1}{1984} \right) - \left(\frac{1}{1985} + \dots + \frac{1}{2000} \right) \right] \\
 &= \frac{1}{16} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{16} - \frac{1}{1985} - \frac{1}{1986} - \dots - \frac{1}{2000} \right)
 \end{aligned}$$

Từ đó suy ra $A = B$.

Câu 13. Chứng minh rằng $G = \frac{1}{4^2} + \frac{1}{6^2} + \frac{1}{8^2} + \dots + \frac{1}{(2n)^2} < 4$

HD:

$$\begin{cases} \frac{1}{4^2} = \frac{1}{(2 \cdot 2)^2} = \frac{1}{4} \cdot \frac{1}{2^2} \\ \frac{1}{6^2} = \frac{1}{(2 \cdot 3)^2} = \frac{1}{4} \cdot \frac{1}{3^2} \\ \dots \\ \frac{1}{(2n)^2} = \frac{1}{(2n)^2} = \frac{1}{4} \cdot \frac{1}{n^2} \end{cases}$$

$$\Rightarrow G = \frac{1}{4} \times \left(\frac{1}{2^2} + \dots + \frac{1}{n^2} \right) \Rightarrow G < \frac{1}{4}$$

CA 2

Câu 10. Tính tổng $S = 1.2 + 2.5 + 3.8 + \dots + n(3n-1)$

HD:

Xét số hạng tổng quát của tổng $k(3k-1) = 3k^2 - k$.

Khi ấy, ta có:

$$S = 1.2 + 2.5 + 3.8 + \dots + n(3n-1)$$

$$S = (3.1^2 - 1) + (3.2^2 - 2) + \dots + (3.n^2 - n)$$

$$S = 3(1^2 + 2^2 + \dots + n^2) - (1 + 2 + 3 + \dots + n)$$

$$S = 3 \cdot \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$S = \frac{n(n+1).2n}{2}$$

$$S = n^2(n+1)$$

Câu 12. Tính tổng $S = 1.2^2 + 2.3^2 + \dots + n(n+1)^2$

HD:

Xét số hạng tổng quát $k(k+1)^2 = k(k+1)[(k+2)-1] = k(k+1)(k+2) - k(k+1)$, $k \in \mathbb{N}$.

Khi đó:

$$S = 1.2^2 + 2.3^2 + \dots + n(n+1)^2$$

$$S = (1.2.3 - 1.2) + (2.3.4 - 2.3) + (3.4.5 - 3.4) + \dots + [n(n+1)(n+2) - n(n+1)]$$

$$S = [1.2.3 + 2.3.4 + 3.4.5 + \dots + n(n+1)(n+2)] - [1.2 + 2.3 + \dots + n(n+1)]$$

$$S = \frac{n(n+1)(n+2)(n+3)}{4} - \frac{n(n+1)(n+2)}{3}$$

$$S = n(n+1)(n+2) \left(\frac{n+3}{4} - \frac{1}{3} \right)$$

$$S = \frac{n(n+1)(n+2)(3n+5)}{12}$$